

# RSM WORKSHOP 2010

## **RSM Dynamics**

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*NOAA/NWS/NCEP/EMC*

# Contents

- Review the dynamic basis we had before
- Elements of GSM/RSM dynamic core
- Recent implementations on semi-Lagrangian
- The extended work on semi-Lagrangian

# Role of Dycore in Model

- **Contains the primitive fluid dynamics**
  - Equation of state
  - Continuity equation
  - Momentum equation
  - Thermodynamic equation
  - Tracer equation
- **Provides reasonable fluid state**
  - For physics etc
  - At any giving required time
  - “Reasonable” magnitude

# Construct Dycore

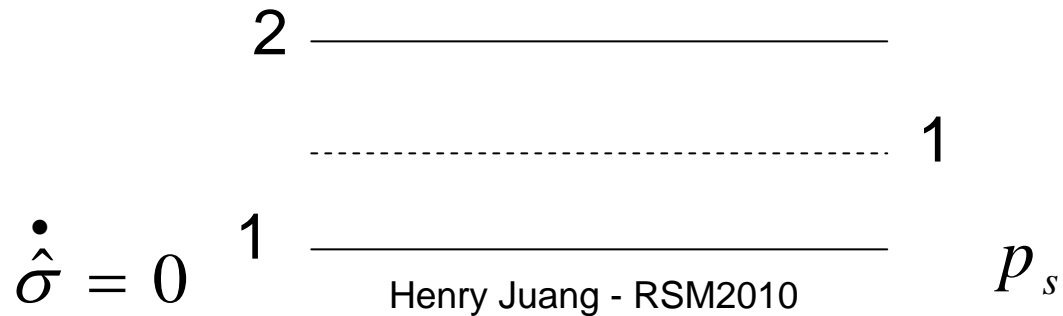
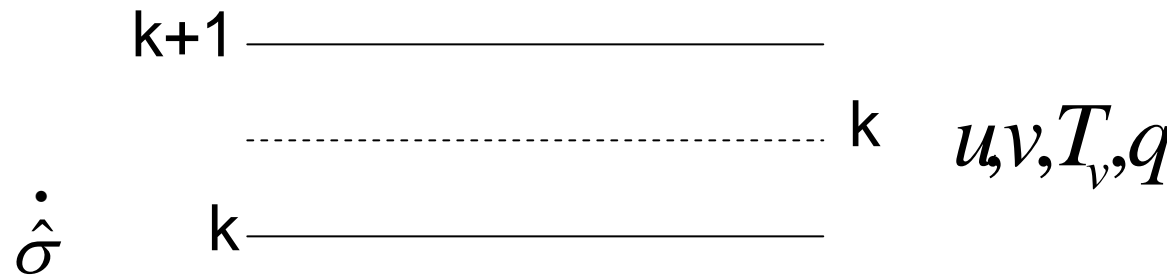
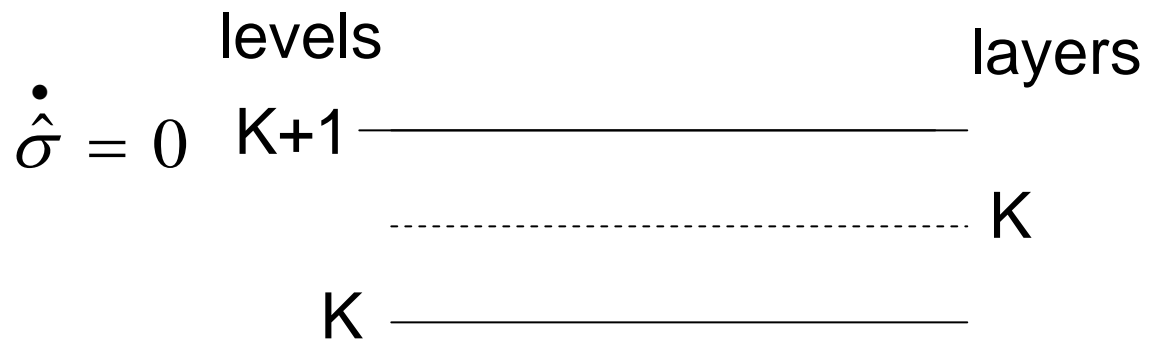
- Keep numerical results exact the same behavior as nature
  - Conservation of Mass, momentum, total energy and potential thermodynamic quantity etc
- Stable integration
  - At any giving required period of time
  - Reasonable magnitude
- Multi Processor Programming

# Horizontal Discretization

- Spectral method
  - Fourier series, FFT, spherical transform
- Nonlinear computation
  - Transform from spectral to grid space
  - Compute nonlinear
  - Transform from grid to spectral space
- Linear computation
  - Compute in spectral space

# Vertical Discretization

- Finite differencing scheme for sigma coordinates
- Use Brown (1974) and Phillips (1974) specification of layer pressure
- Lorenz grid : only vertical motion in grid interface, others all represent mean value of the given layer



# Hydrostatic system in sigma vertical coordinates

$$\begin{aligned}
 \frac{\partial u^*}{\partial t} &= -m^2 u^* \frac{\partial u^*}{\partial x} - m^2 v^* \frac{\partial u^*}{\partial y} - \sigma \frac{\partial u^*}{\partial \sigma} \left[ R_d T_v \frac{\partial Q}{\partial x} - \frac{\partial \Phi}{\partial x} \right] + f_s v^* - E \frac{\partial m^2}{\partial x} \\
 \frac{\partial v^*}{\partial t} &= -m^2 u^* \frac{\partial v^*}{\partial x} - m^2 v^* \frac{\partial v^*}{\partial y} - \sigma \frac{\partial v^*}{\partial \sigma} \left[ R_d T_v \frac{\partial Q}{\partial y} - \frac{\partial \Phi}{\partial y} \right] - f_s u^* - E \frac{\partial m^2}{\partial y} \\
 \frac{\partial T_v}{\partial t} &= -m^2 u^* \frac{\partial T_v}{\partial x} - m^2 v^* \frac{\partial T_v}{\partial y} - \sigma \frac{\partial T_v}{\partial \sigma} + \kappa T_v \left( \frac{\partial Q}{\partial t} + m^2 u^* \frac{\partial Q}{\partial x} + m^2 v^* \frac{\partial Q}{\partial y} + \frac{\partial \sigma}{\partial \sigma} \right) \\
 \frac{\partial q_i}{\partial t} &= -m^2 u^* \frac{\partial q_i}{\partial x} - m^2 v^* \frac{\partial q_i}{\partial y} - \sigma \frac{\partial q_i}{\partial \sigma} \\
 \frac{\partial Q}{\partial t} &= -m^2 u^* \frac{\partial Q}{\partial x} - m^2 v^* \frac{\partial Q}{\partial y} - m^2 \left( \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} \right) - \frac{\partial \sigma}{\partial \sigma}
 \end{aligned}$$

where m is map factor  $E = \frac{1}{2} \left[ (u^*)^2 + (v^*)^2 \right]$  and  $Q = \ln p_s$



# Conservation

- Discretized system has the same characteristics as continuum system
- The characteristics are most conservations
- The method to satisfy this condition
  - Make two discretized forms with the characteristics from one form to another form

# Mass weighted vertically integration of PGF

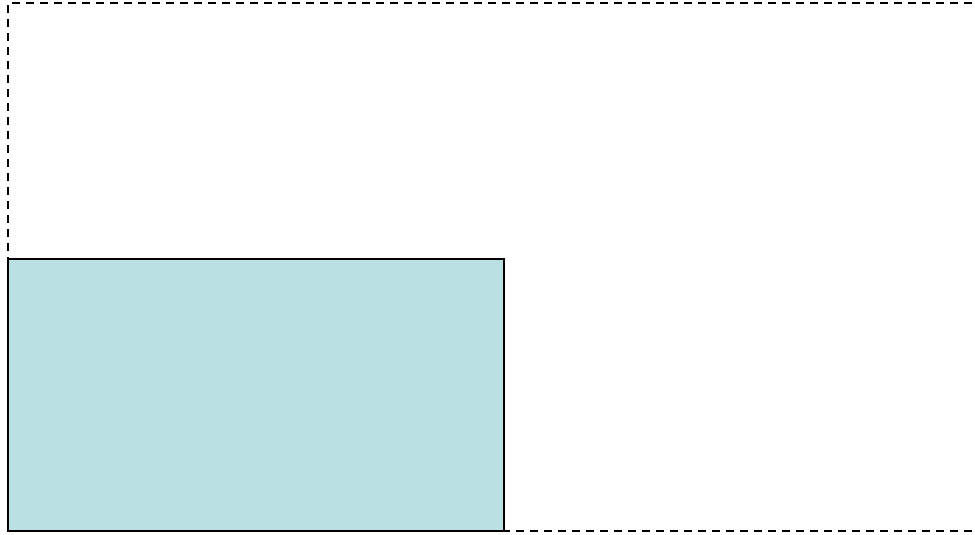
$$\begin{aligned}
 \int_0^1 p_s (\nabla \Phi + R_d T_v \nabla Q) d\sigma &= \int_0^1 \nabla (p_s \Phi) d\sigma - \int_0^1 (p_s \Phi \nabla Q - p_s R_d T \nabla Q) d\sigma \\
 &= \int_0^1 \nabla (p_s \Phi) d\sigma - \int_0^1 p_s \left( \Phi \nabla \frac{\partial \sigma Q}{\partial \sigma} + \frac{\partial \Phi}{\partial \sigma} \nabla \sigma Q \right) d\sigma \\
 &= \nabla \int_0^1 (p_s \Phi) d\sigma - p_s \int_0^1 \frac{\partial \Phi \nabla \sigma Q}{\partial \sigma} d\sigma \\
 &= \nabla \int_0^1 (p_s \Phi) d\sigma - p_s \Phi_s \nabla Q
 \end{aligned}$$

It results a simple relation as  $\int_0^1 (\Phi - R_d T) d\sigma = \Phi_s$

expandable as  $\Phi_s + \sum_{j=1}^K (R_d T_v)_j \Delta \sigma_j = \sum_{j=1}^K \Phi_j \Delta \sigma_j = \Phi_1 + \sum_{k=1}^{K-1} (\Phi_{j+1} - \Phi_j) \hat{\sigma}_{j+1}$

# Stability

- Boundary treatment to remove the noise from the inconsistency between inner and outer grids
- Using spatial and/or temporal filters to control high frequency in space and/or in time
- Using small time step to control Numerical instability due to nonlinear interaction, such as advection term



$$A' = A - A_G$$

Extend value of  $A'$  by sine or cosine shape  
let  $A'$  by spectral transform

Let  $A'$  at the boundary to be zero or relax to be zero

# lateral boundary relaxation

- Explicit

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{F} - \lambda(\mathbf{A}^{n-1} - \mathbf{A}_b^{n-1})$$

$$\frac{\partial \mathbf{A}'}{\partial t} = \mathbf{F} - \frac{\partial \mathbf{A}_b}{\partial t} - \lambda(\mathbf{A}^{n-1} - \mathbf{A}_b^{n-1})$$

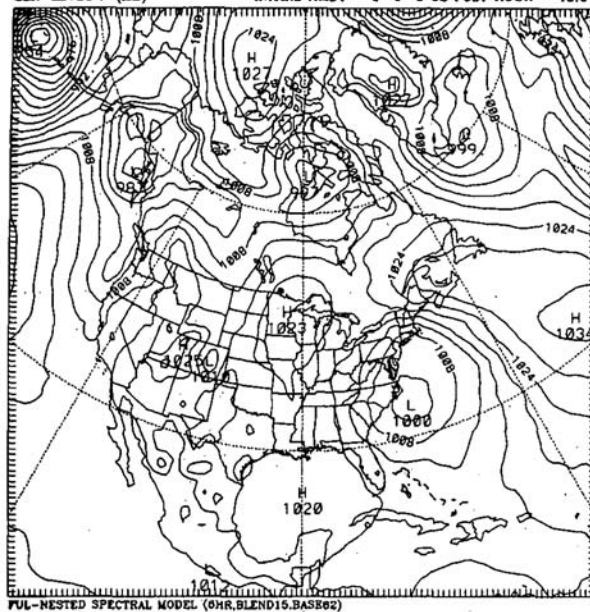
# lateral boundary relaxation

- Implicit, time-splitting

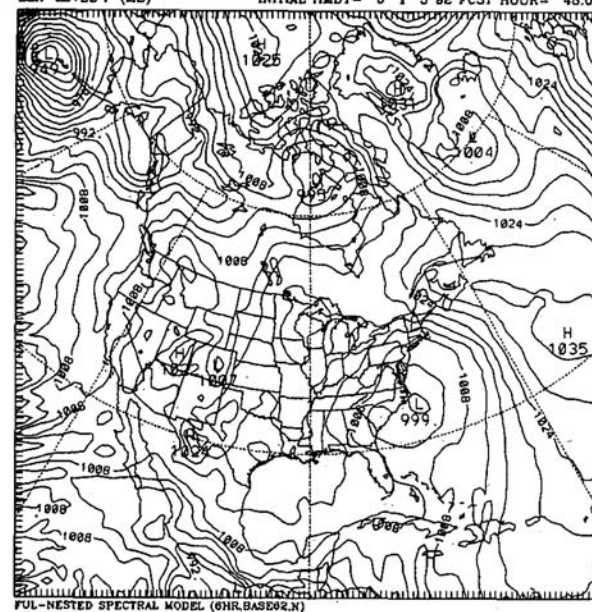
$$\frac{\partial A}{\partial t} = F - \lambda(A^{n+1} - A_b^{n+1})$$

$$A^{n+1} = A^{n-1} + 2\Delta t \frac{\partial A}{\partial t}$$

(a) SEA-LEVEL P (MB) INITIAL HMDY= 0 1 3 92 FCST HOUR= 48.0



(c) SEA-LEVEL P (MB) INITIAL HMDY= 0 1 3 92 FCST HOUR= 48.0



(b) SEA-LEVEL P (MB) INITIAL HMDY= 0 1 3 92 FCST HOUR= 48.0

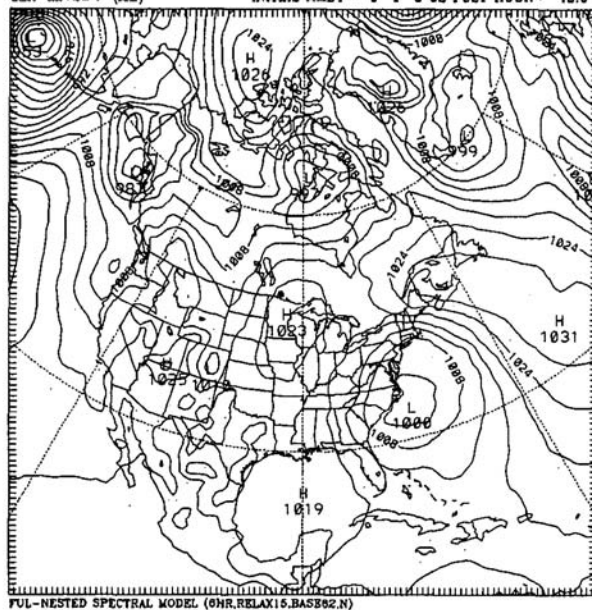


FIG. 4. Regional forecasts of sea level pressure (contour interval—4 mb) after a 48-h prediction with (a) blending and relaxation, (b) relaxation only, and (c) no blending and no relaxation, starting at 0000 UTC 3 January 1992.

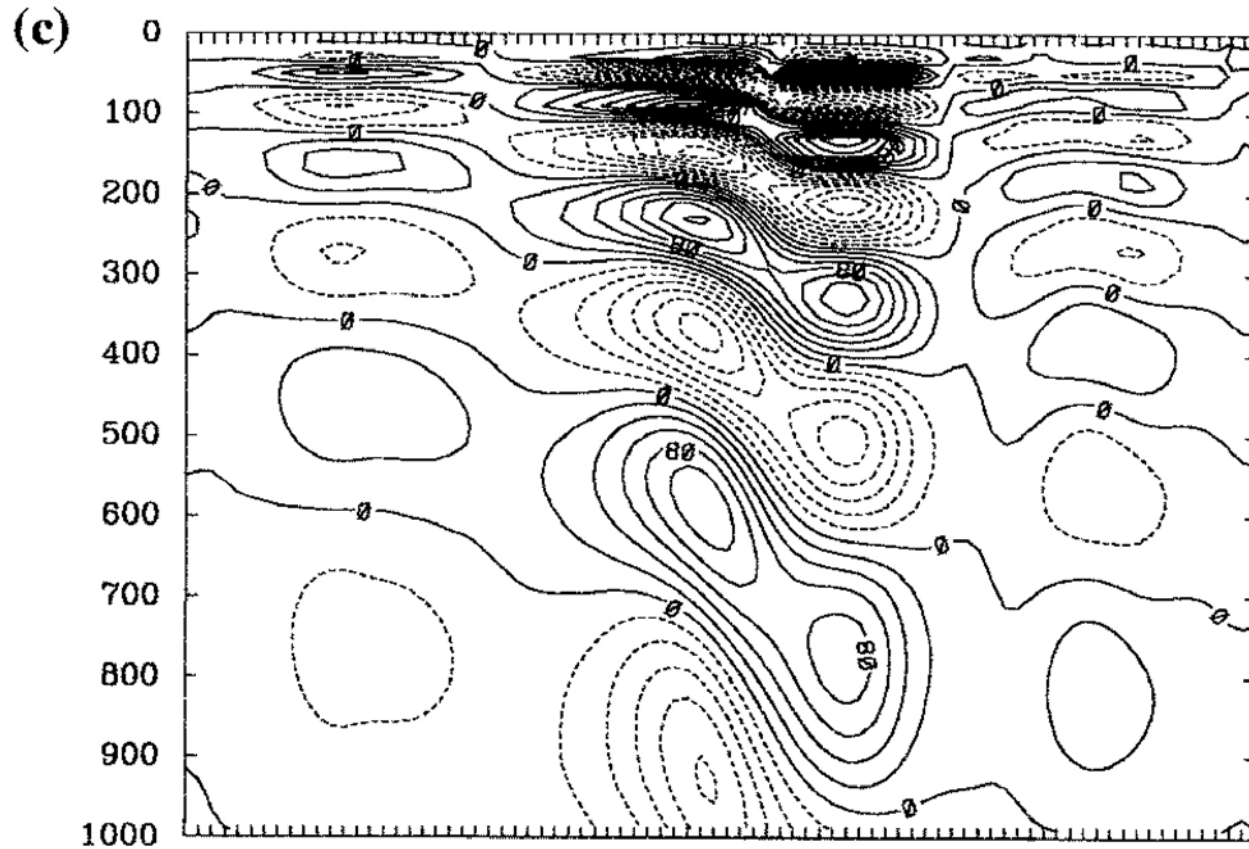
# top layers relaxation

- optional for RSM
- may be need for theoretical or idealized cases.
- it can be written as

$$\frac{\partial A'}{\partial t} = F' - \frac{1}{n\Delta t} \text{Max}\left[\frac{\sigma_m - \sigma_k}{\sigma_m - \sigma_t}, 0\right] A'$$

where  $\sigma_m$  is the layer starting the relaxation,  $\sigma_t$  is the model top layer,  $\sigma_k$  is the any layer k.





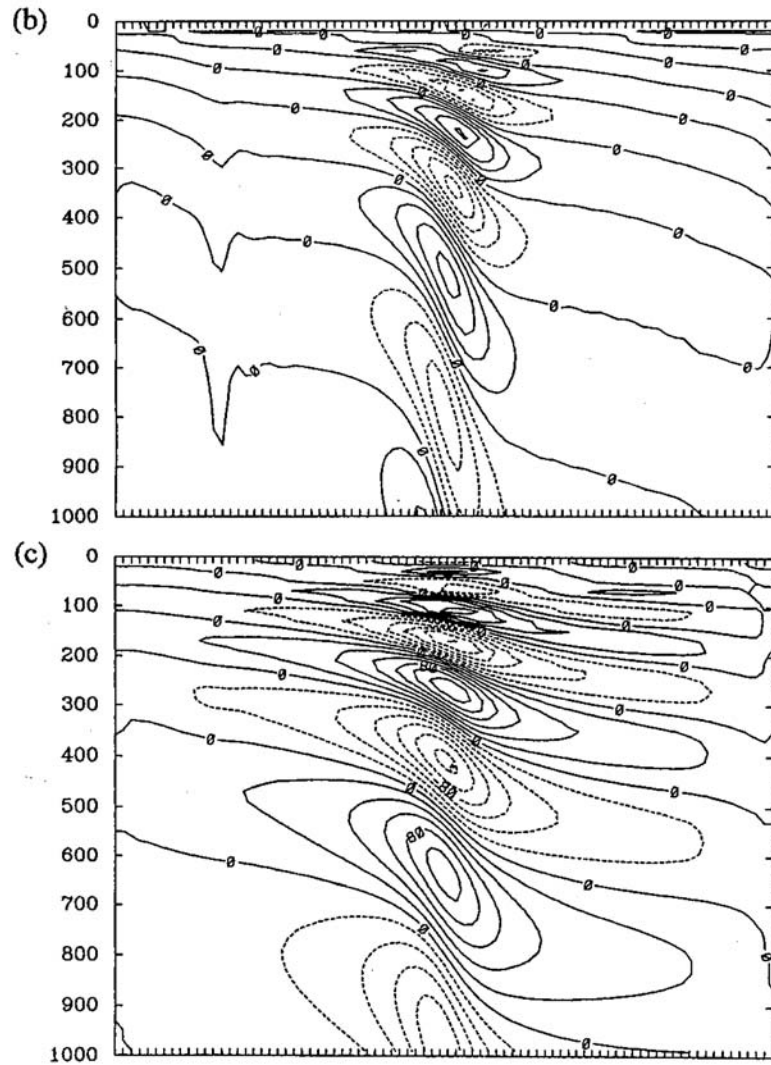


FIG. 8. The same as in Fig. 7 except from the current revised fully internally evolved hydrostatic coordinates of MSM with top-layer relaxation.

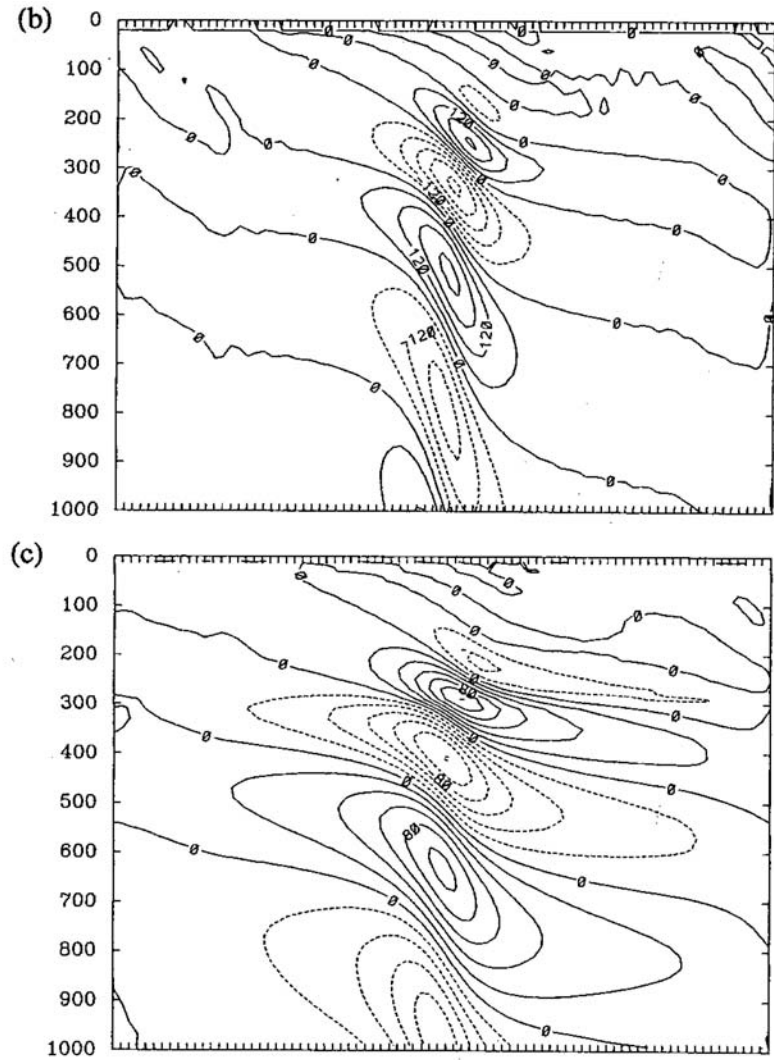
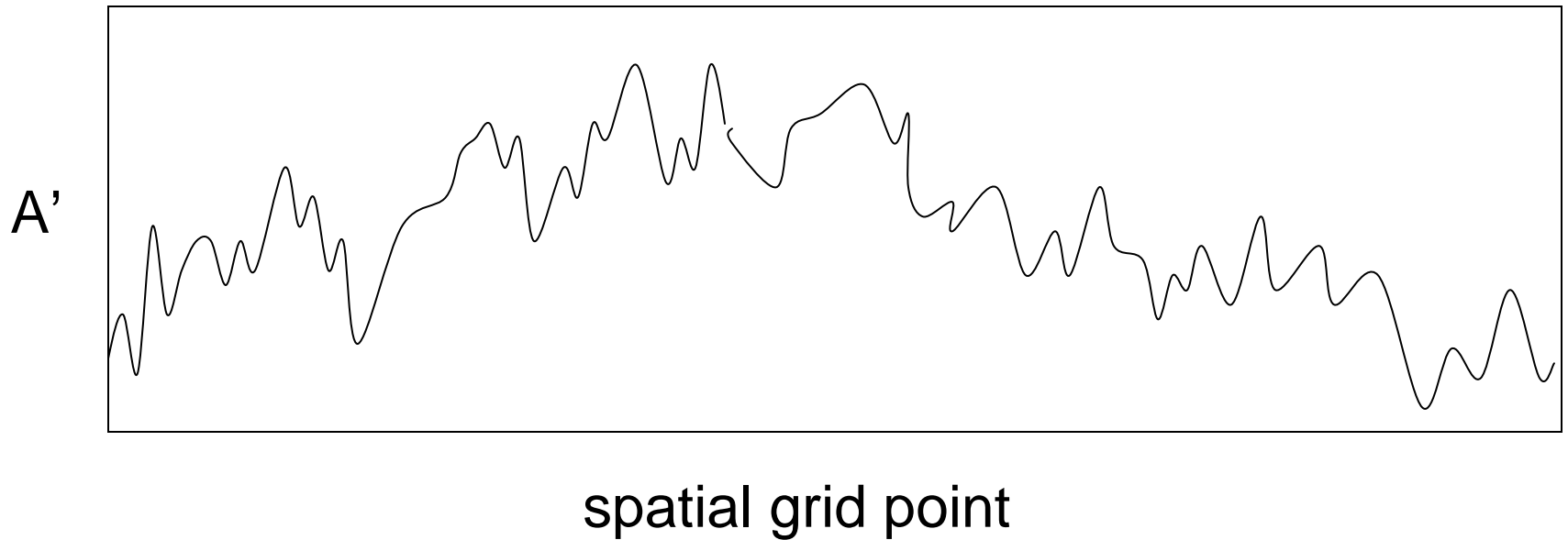


FIG. 9. The same as in Fig. 8 except from the previous version with time-independent coordinates (J92).



### Domain filter

transforming grid-point values into spectral coefficient  
and remove high frequency

### Local filterer

smoothing by removing high frequency through the  
nearby grid points

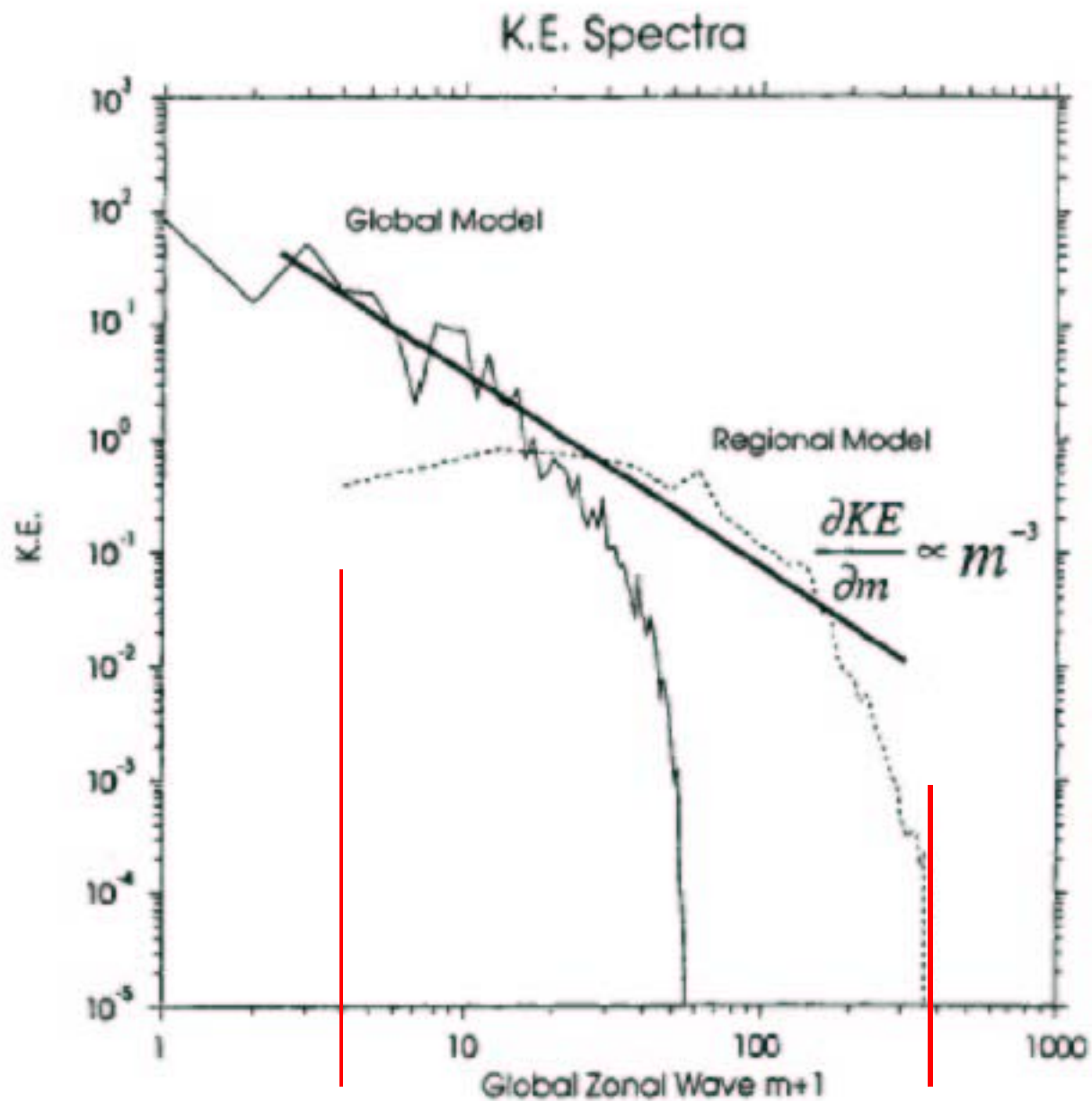


FIG. 2. Regional (dotted line) and global (thin solid line) model kinetic energy spectra. Courtesy of Chen et al. (1999).

# horizontal diffusion

- Diffusion on the perturbation

$$\frac{\partial A}{\partial t} = F - k \nabla^4 A$$

$$\frac{\partial A}{\partial t} = F - \frac{1}{\mu \Delta t} \left[ \frac{\left(\frac{\pi m}{L_x}\right)^2 + \left(\frac{\pi n}{L_y}\right)^2}{\left(\frac{\pi M}{L_x}\right)^2 + \left(\frac{\pi N}{L_y}\right)^2} \right]^2 (A' + A_b)$$

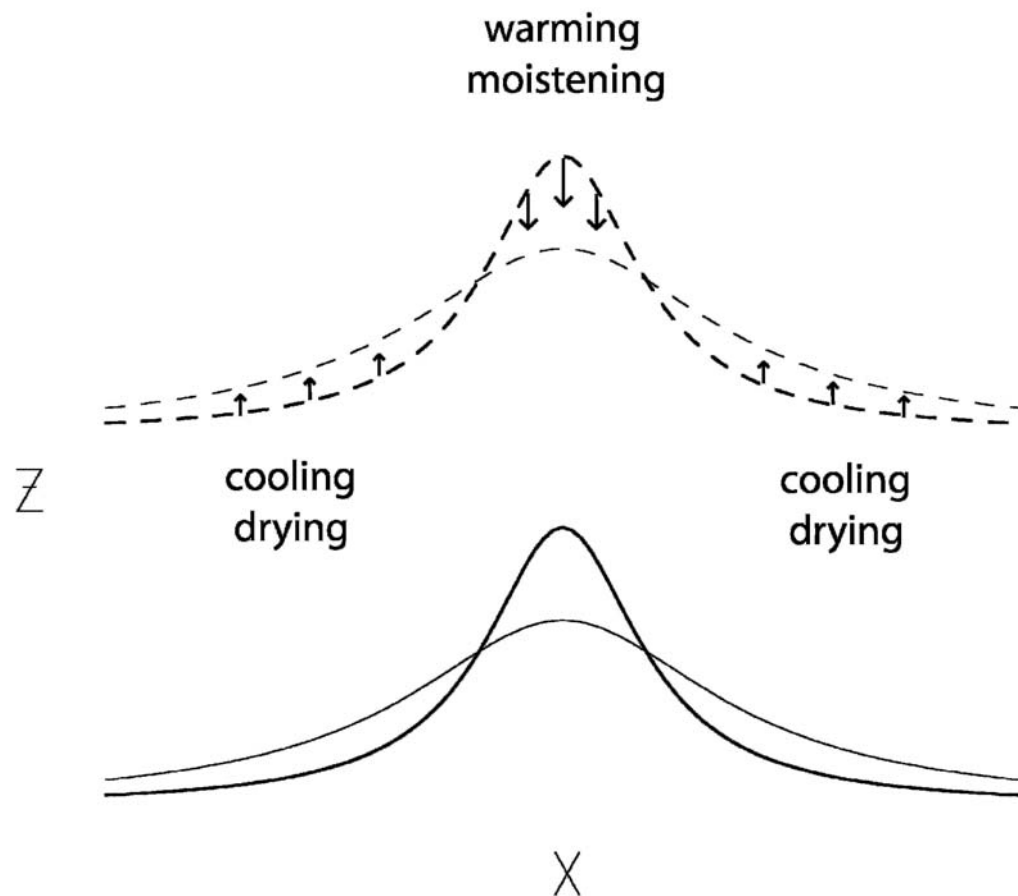
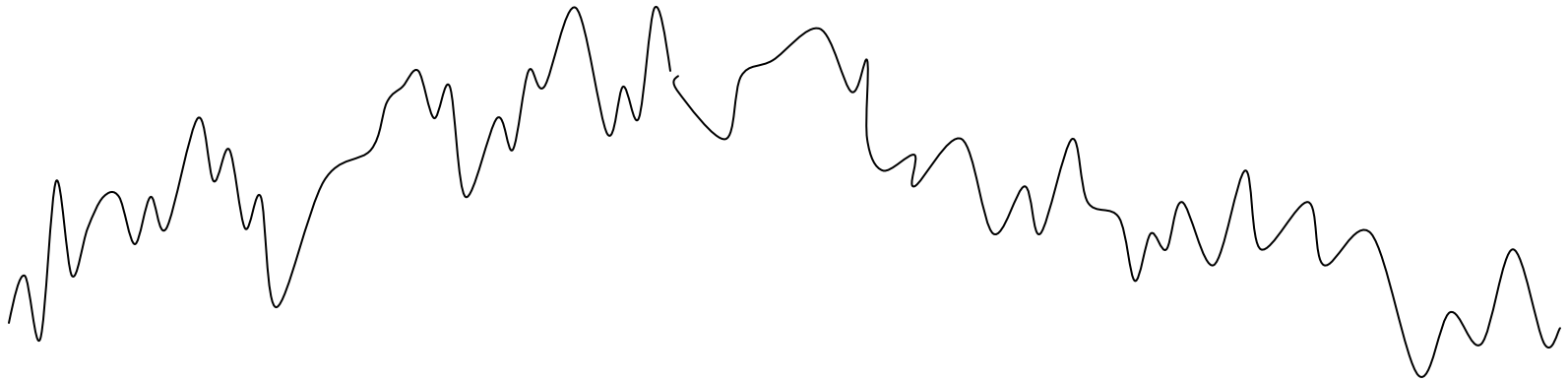


FIG. 1. A schematic plot to show the differences in height of the same sigma layers due to the model terrain differences between the outmost coarse-resolution model (light solid and dashed curves) and inner fine-resolution model (heavy solid and dashed curves). The solid curves indicate the model terrain heights, and the dashed curves indicate the model layer heights. The arrows indicate the direction of the temperature changes after applying horizontal diffusion on sigma surfaces; i.e., the temperature at heavy dashed curve will be relaxed to the value at the light dashed curve.



temporal grid point

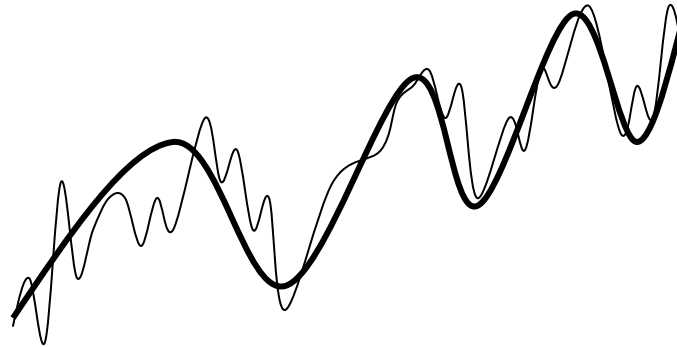
## Filtering the values

transforming grid-point values into spectral coefficient  
and remove high frequency

## Forcing filter

smoothing by removing high frequency through the  
nearby grid points

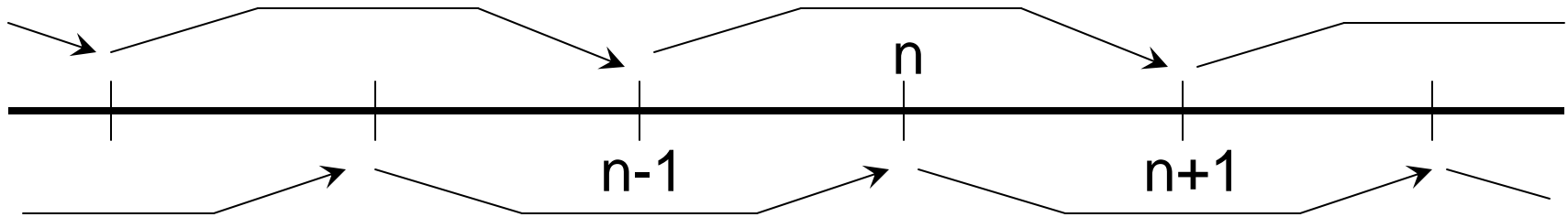
# forward digital initialization



- optional
- for example, run up to 6 hour
- spectral transform in time
- filter to have value at hour 3
- continue integration from hour 3
- to turn on, set **INIHREG=6** in run script
- to turn off, set **INIHREG=0**.

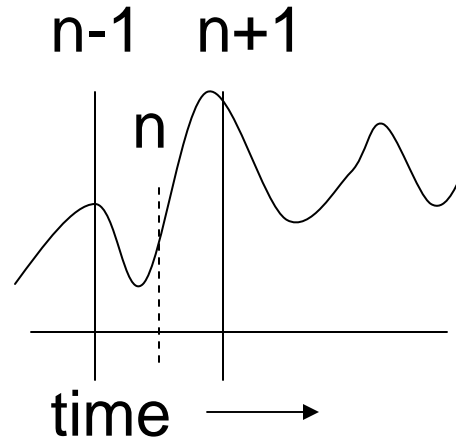


# time filter for 3 time levels



- Asselin (1972)
- three time level time scheme

$$A^n = A_*^n + \epsilon (A_*^{n+1} - 2A_*^n + A^{n-1})$$



$$\frac{\partial A}{\partial t} = F$$

Filter with forcing

$$A^{n+1} = A^{n-1} + F^n 2\Delta t$$

$$A^{n+1} = A^{n-1} + (F^{n+1} + F^{n-1})\Delta t$$

$$F^n = L^n + N^n$$

$$= \frac{1}{2}(L^{n+1} + L^{n-1}) + N^n$$

Semi-implicit

# A possible next version

- Generalized vertical coordinates (in GFS)
  - sigma
  - Hybrid sigma-pressure
  - Hybrid sigma-theta
- Accuracy, conservation and time saving
  - Enthalpy as thermodynamic variable (in GFS)
  - Non-iteration Dimensional-split semi-Lagrangian (in GFS)
- Explicitly solving gravity and acoustic waves by NDSL with Riemann solver (under testing)

# A specific form for generalized hybrid

$$\hat{p}_k = \hat{A}_k + \hat{B}_k p_s + \hat{C}_k \left( \hat{T}_{vk} / \hat{T}_{0k} \right)^{C_p / R_d}$$

where

$$\hat{A}_{K+1} = \hat{B}_{K+1} = \hat{C}_{K+1} = 0$$

$$\hat{A}_1 = \hat{C}_1 = 0$$

$$\hat{B}_1 = 1$$

For sigma-pressure, let  $\hat{C}_k = 0$  for all levels

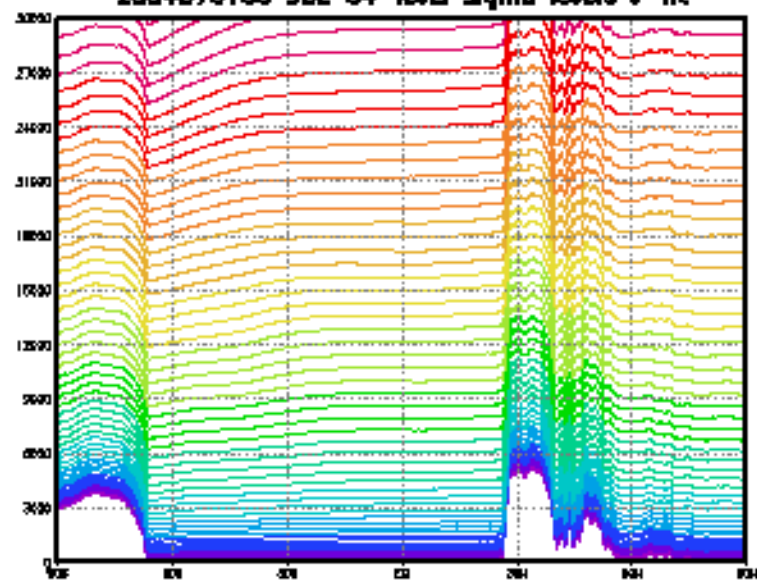
$\hat{B}_k$  - decrease from 1 to 0 at low layers

$\hat{A}_k$  - increase from 0 to be pressure where  $B=0$ , then decrease to zero.

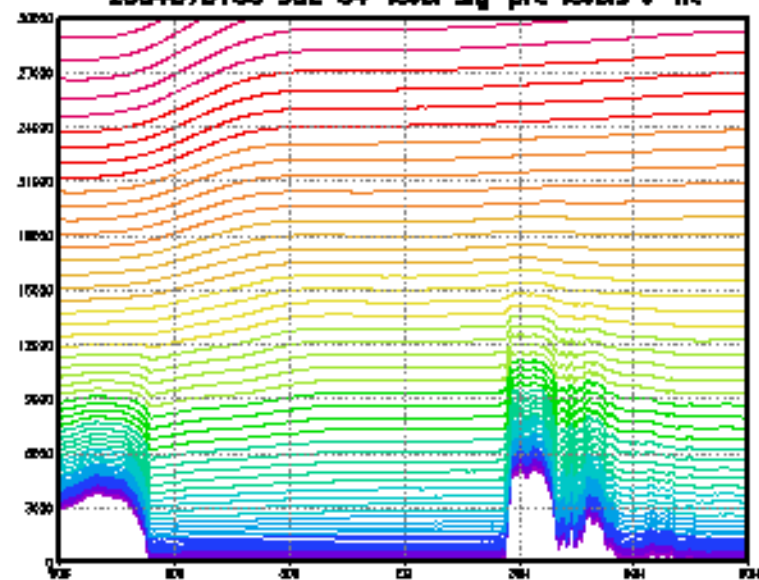
For sigma-theta, let  $\hat{A}_k = 0$  for all levels

$\hat{B}_k$  works the same, and  $\hat{C}_k$  works as  $\hat{A}_k$  in sigma-p

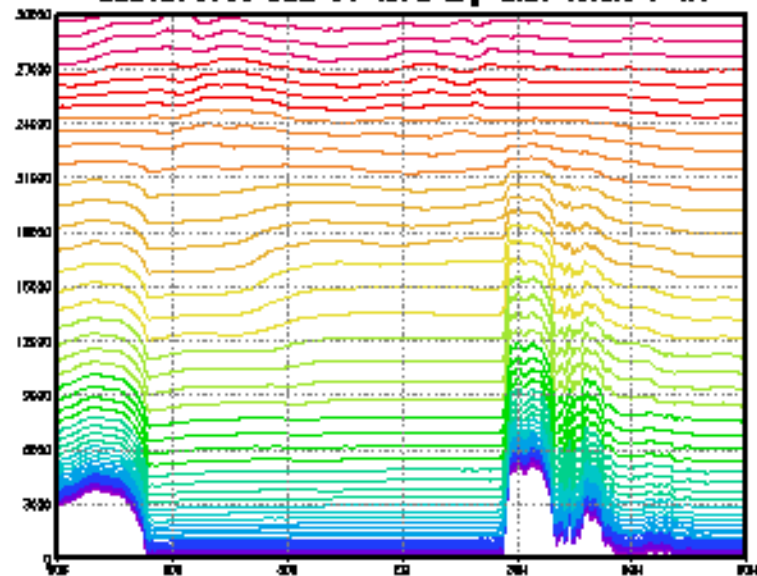
2004070100 90E 64-level sigma levels v-ht



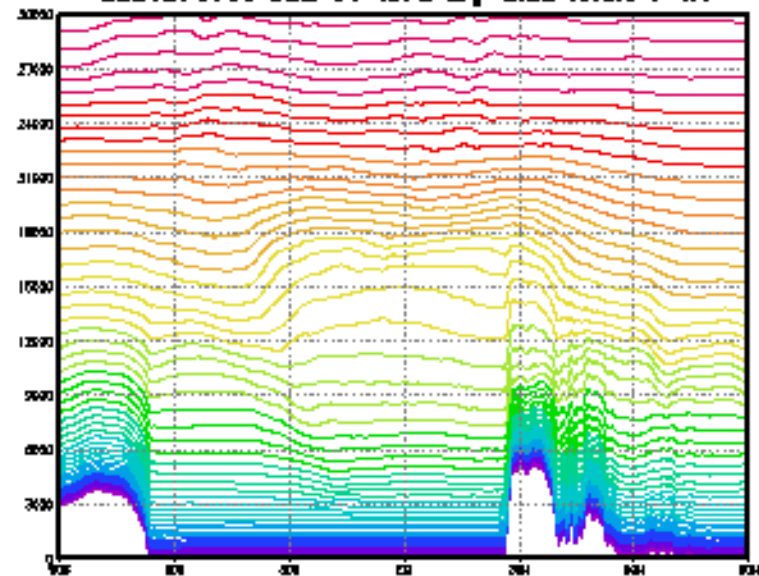
2004070100 90E 64-level sig-pre levels v-ht



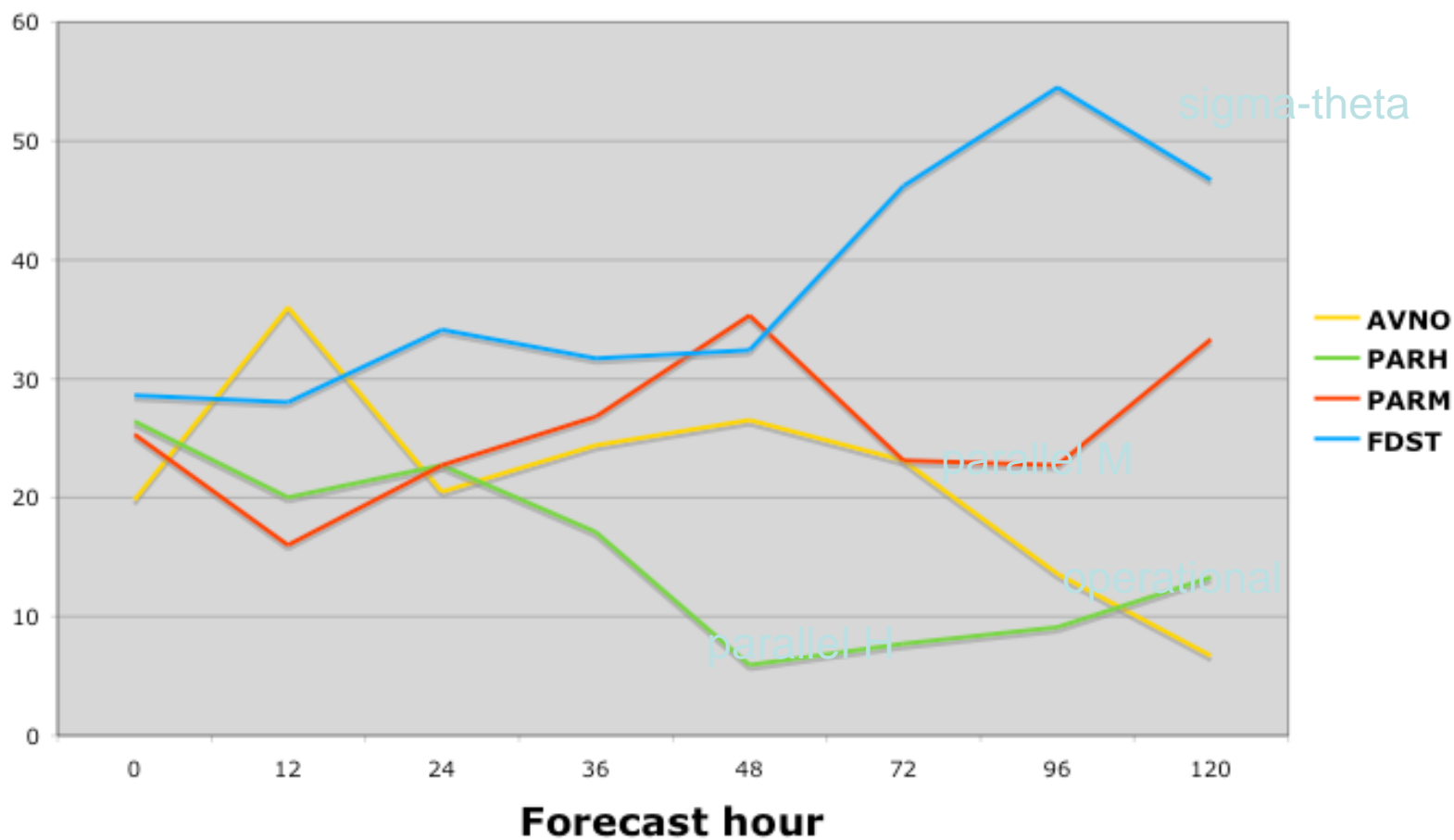
2004070100 90E 64-level sig-the1 levels v-ht



2004070100 90E 64-level sig-the2 levels v-ht



## Frequency of Superior Performance (%)



2005 hurricane season

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Numerical instability due to advection by the CFL condition

$$\frac{\partial q}{\partial t} = -u \frac{\partial q}{\partial x}$$

Let  $q = Q(z)e^{-(x+wt)i}$

$$\Delta x - u\Delta t > 0$$

$$u\Delta t / \Delta x < 1$$

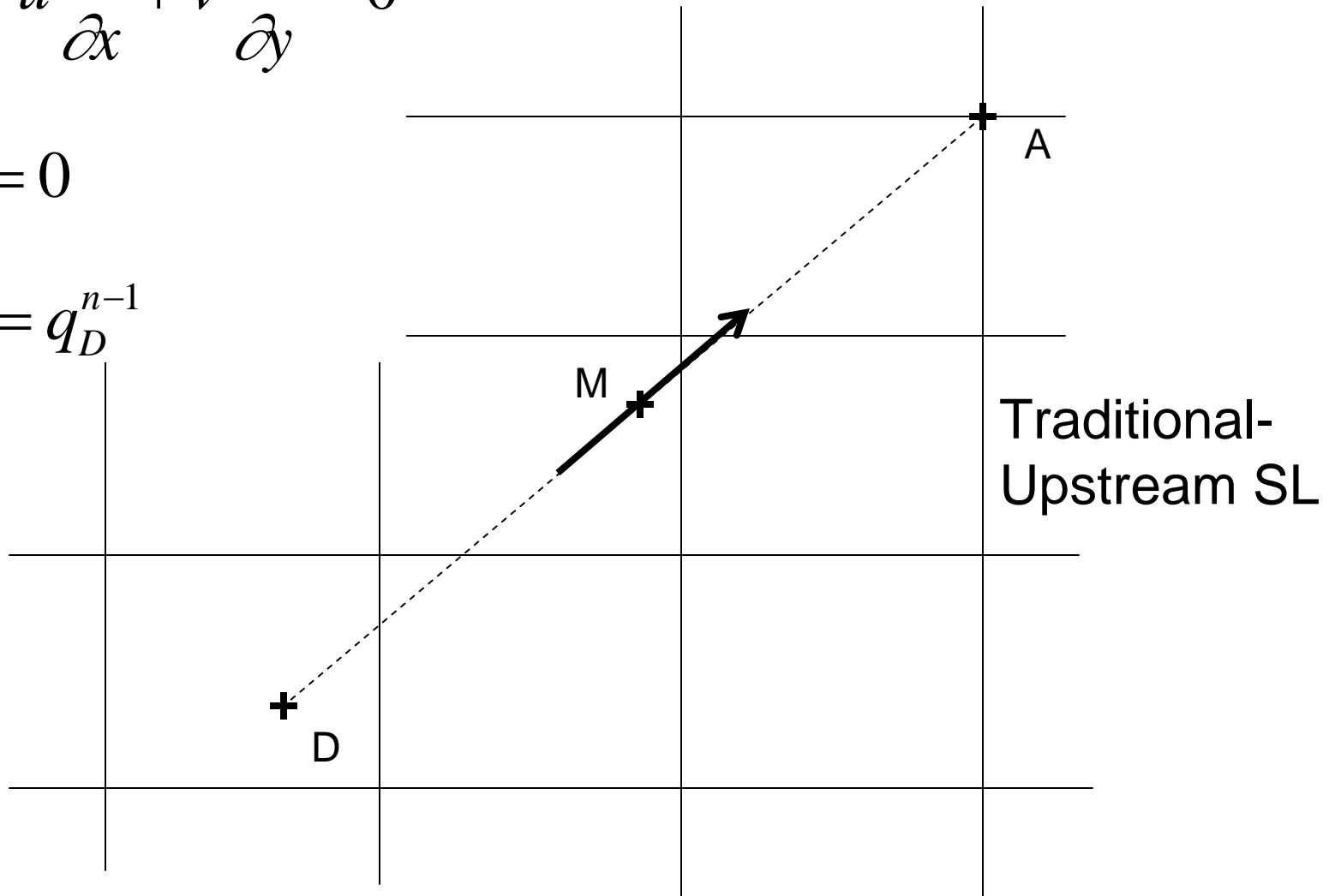
i.e  $u=200\text{m/s}$   $dx=10000\text{m}$   $dt < 50 \text{ sec}$

But if we solve by  $\frac{dq}{dt} = 0$  then unconditional stable

$$\frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x} + v \frac{\partial q}{\partial y} = 0$$

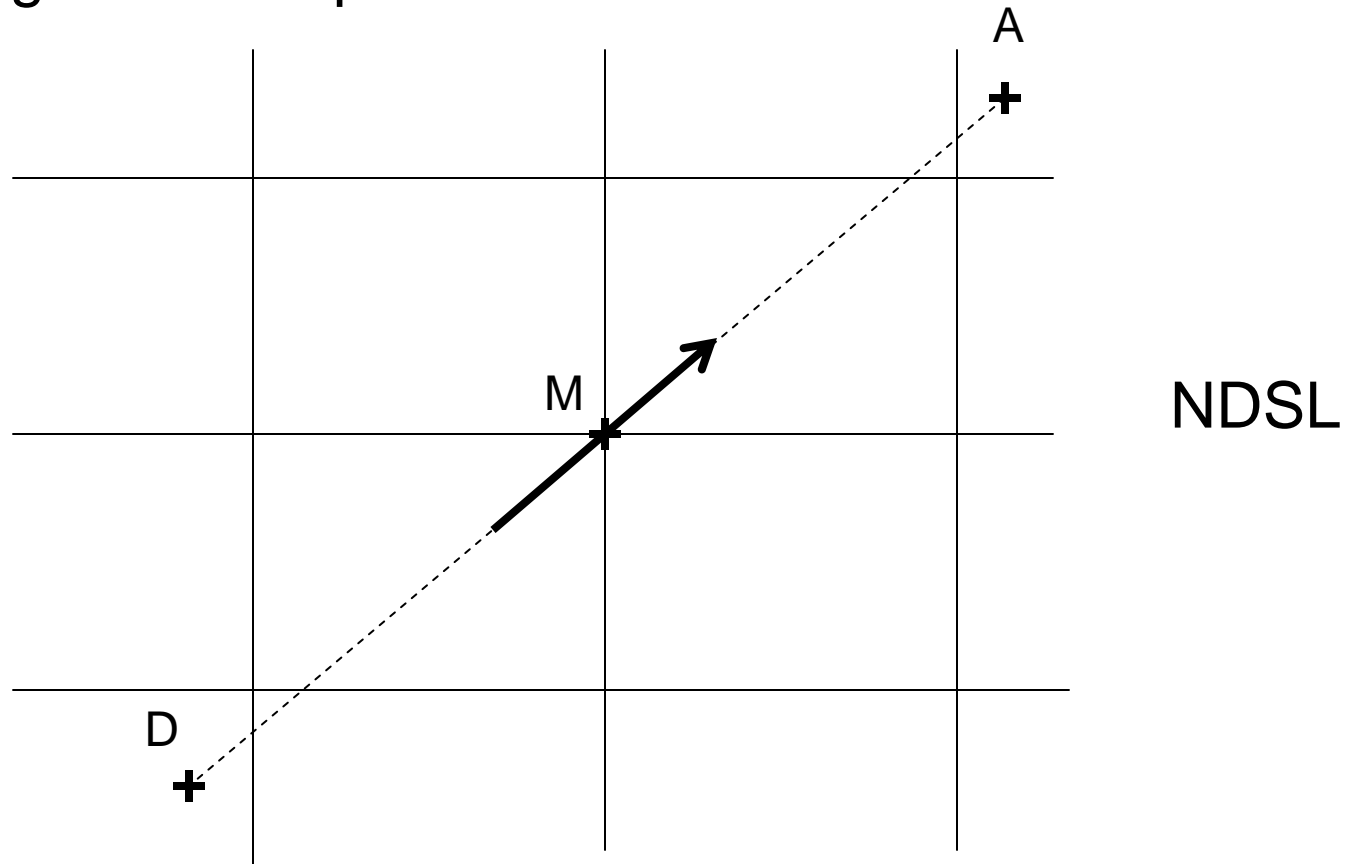
$$\frac{Dq}{Dt} = 0$$

$$q_A^{n+1} = q_D^{n-1}$$





Starting from mid-point



No guessing and no iteration

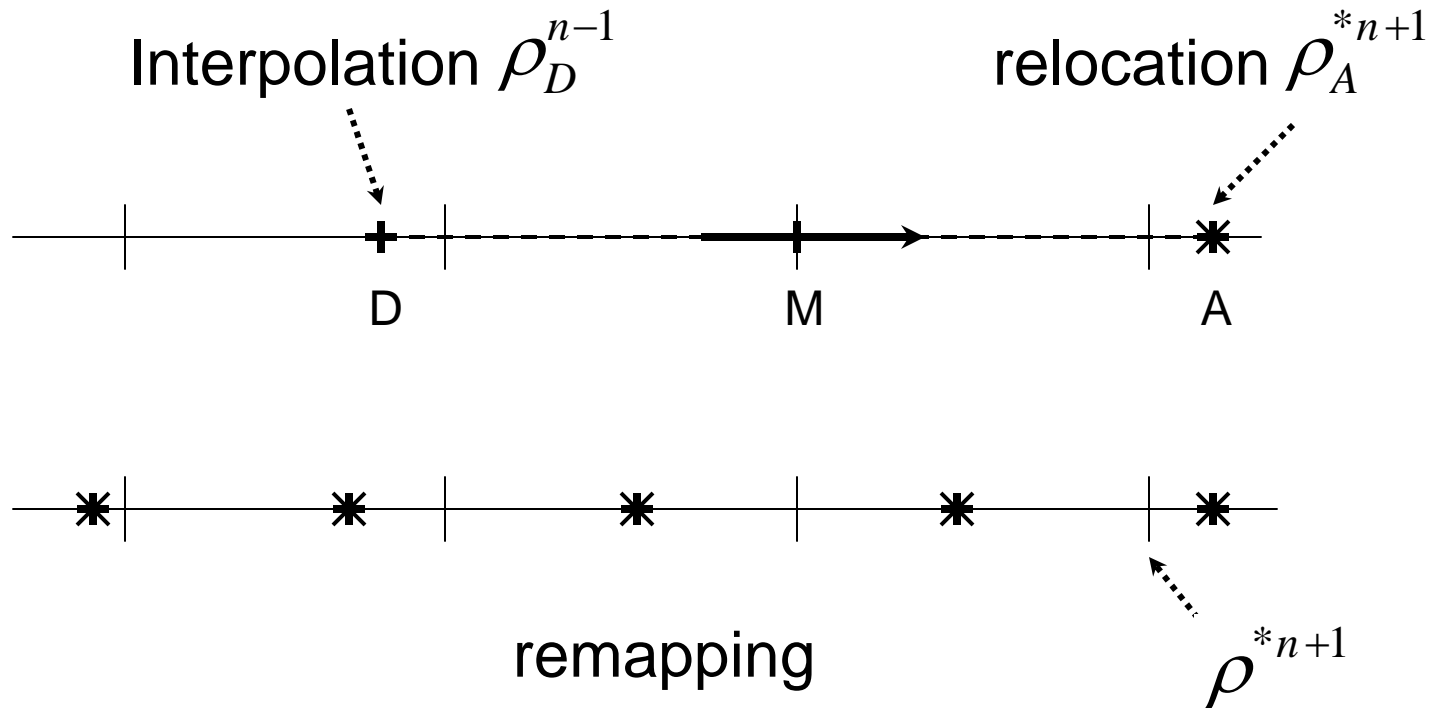
but one 2-D interpolation and one 2-D remapping

Instead doing following

$$X_D = X_M - U_M \Delta t$$

$$X_A = X_M + U_M \Delta t$$

$$\rho_A^{*n+1} = \rho_D^{n-1}$$



For mass conservation, let's start from continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0$$

$$\left\langle \left( \frac{\partial \rho}{\partial t} \right)_{X\text{-direction}} + \frac{\partial \rho u}{\partial x} \right\rangle + \left\langle \left( \frac{\partial \rho}{\partial t} \right)_{Y\text{-direction}} + \frac{\partial \rho v}{\partial y} \right\rangle = 0$$

Consider 1-D and rewrite it in advection form, we have

$$\left( \frac{\partial \rho}{\partial t} \right)_{X\text{-direction}} + u \frac{\partial \rho}{\partial x} = -\rho \frac{\partial u}{\partial x}$$

Advection form is for semi-Lagrangian,  
but it is not conserved if divergence is treated as force at mid-point,  
So divergence term should be treated with advection

Divergence term in Lagrangian sense is the change of the volume if mass is conserved, so we can write divergence form as

$$\left( \frac{\partial u}{\partial x} \right)_{Lagrangian\_sense} = \frac{1}{\Delta_x} \frac{d\Delta_x}{dt}$$

Put it into the previous continuity equation, we have

$$\left( \frac{d\rho\Delta_x}{dt} \right)_{X-direction} = 0$$

$$\left( \frac{\partial\rho\Delta_x}{\partial t} \right)_{X-direction} + u \frac{\partial\rho\Delta_x}{\partial x} = 0$$

which can be seen as  $(\rho\Delta_x)_{departure} = (\rho\Delta_x)_{arrival}$

We do

$$X_L^D = X_L^M - U_L^M \Delta t$$

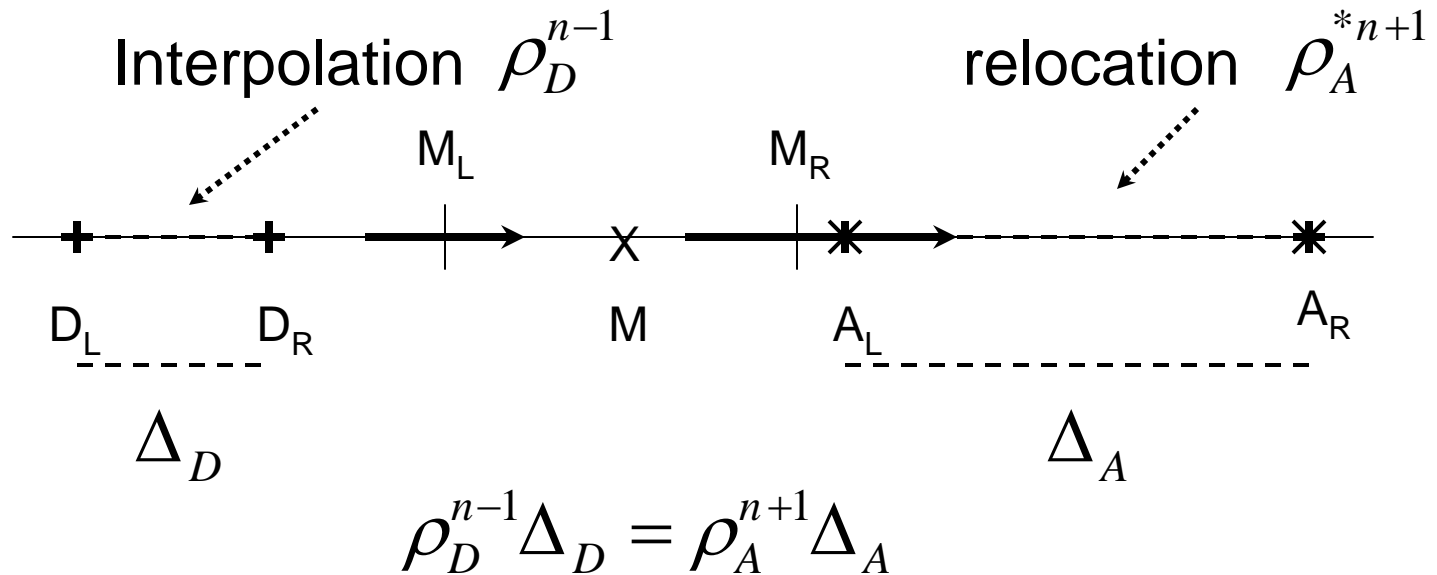
$$X_R^D = X_R^M - U_R^M \Delta t$$

$$\Delta_D = X_R^D - X_L^D$$

$$X_L^A = X_L^M + U_L^M \Delta t$$

$$X_R^A = X_R^M + U_R^M \Delta t$$

$$\Delta_A = X_R^A - X_L^A$$

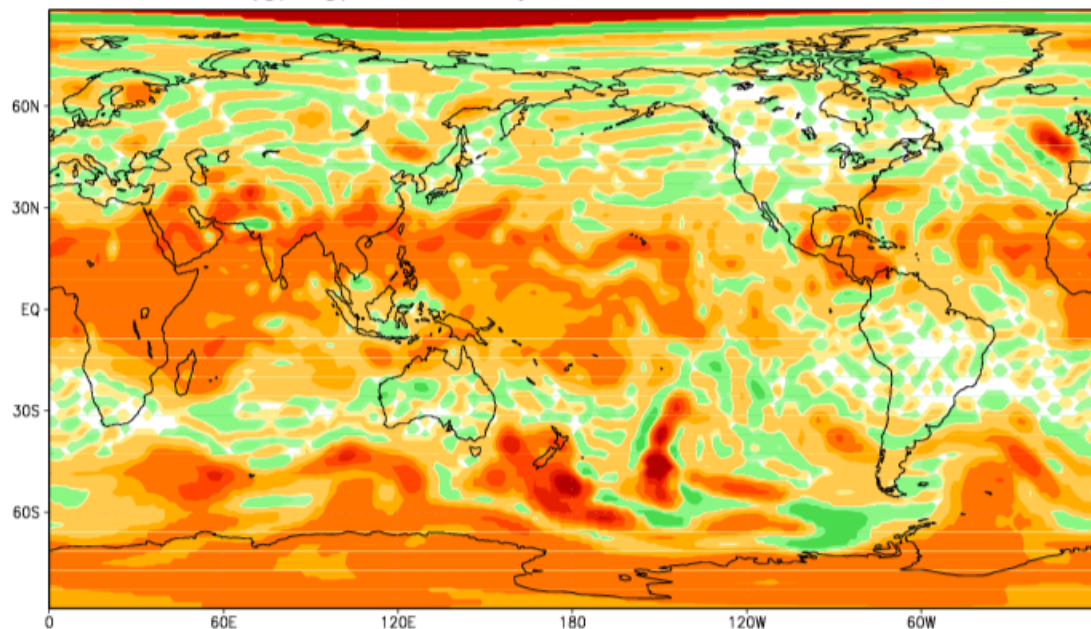


control

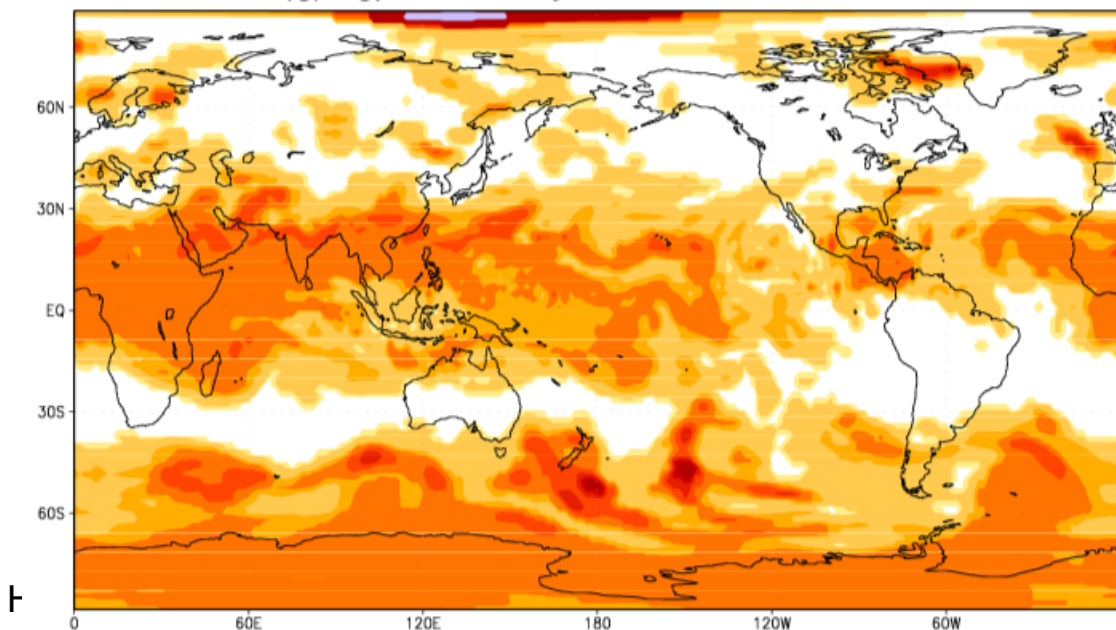
06h fcst specific humidity  
at model layer 40

nislfv

SPFH(g/kg) model layer 40 hour 06 control run



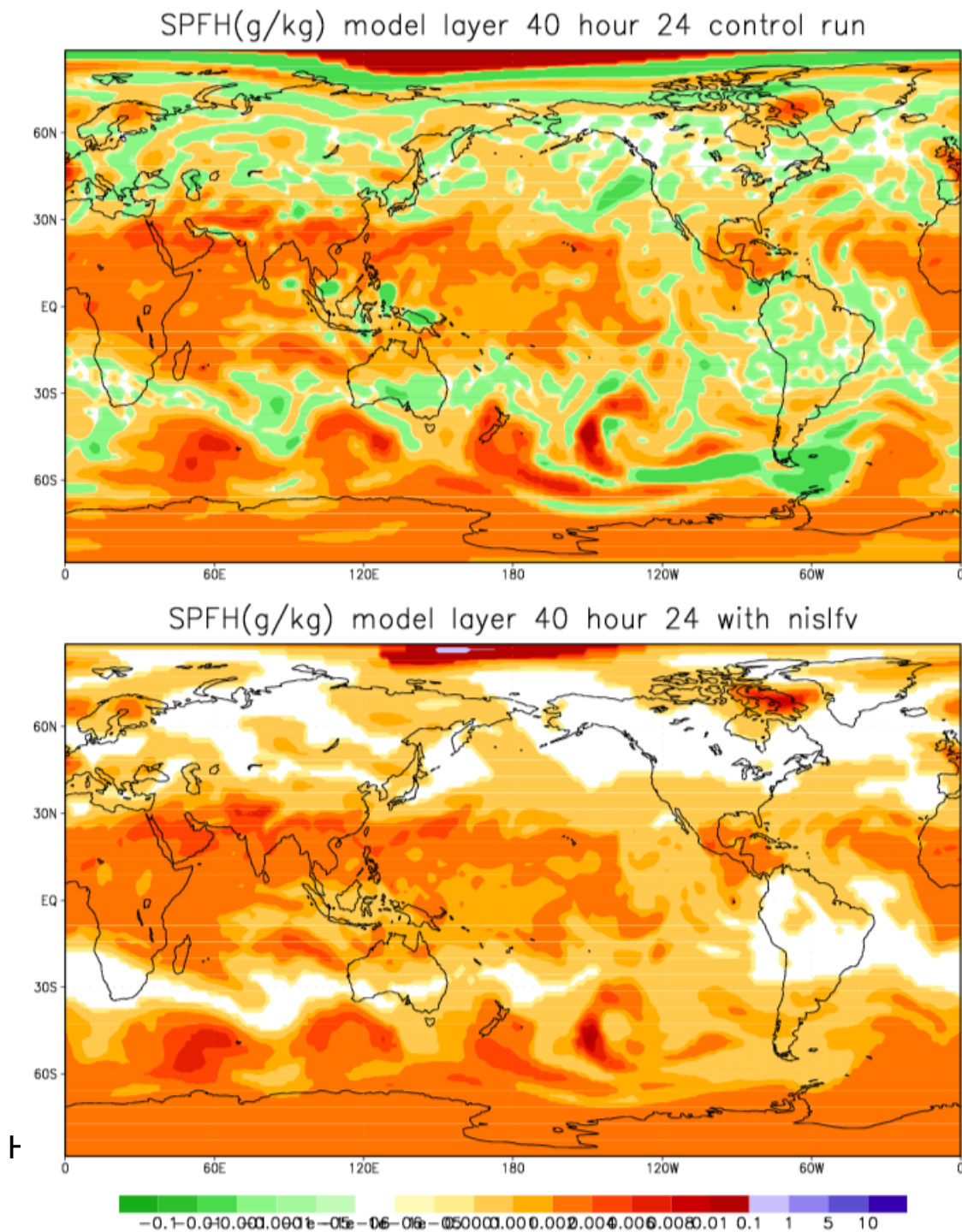
SPFH(g/kg) model layer 40 hour 06 with nislfv



control

# 24h fcst specific humidity at model layer 40

**nislfv**



# 72h fcst specific humidity at model layer 40

H



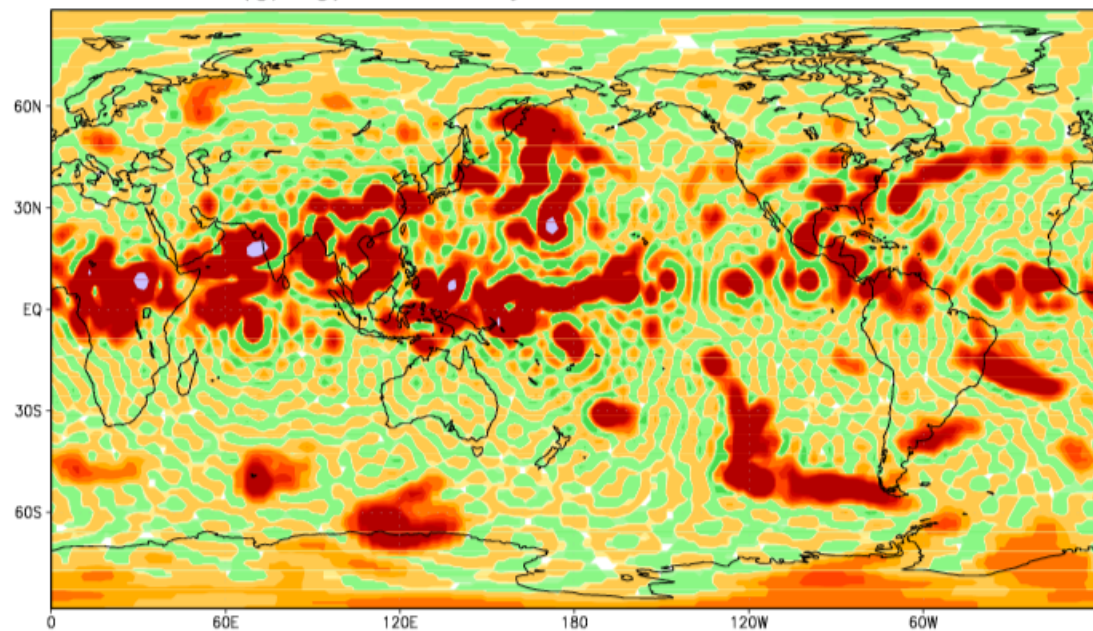


control

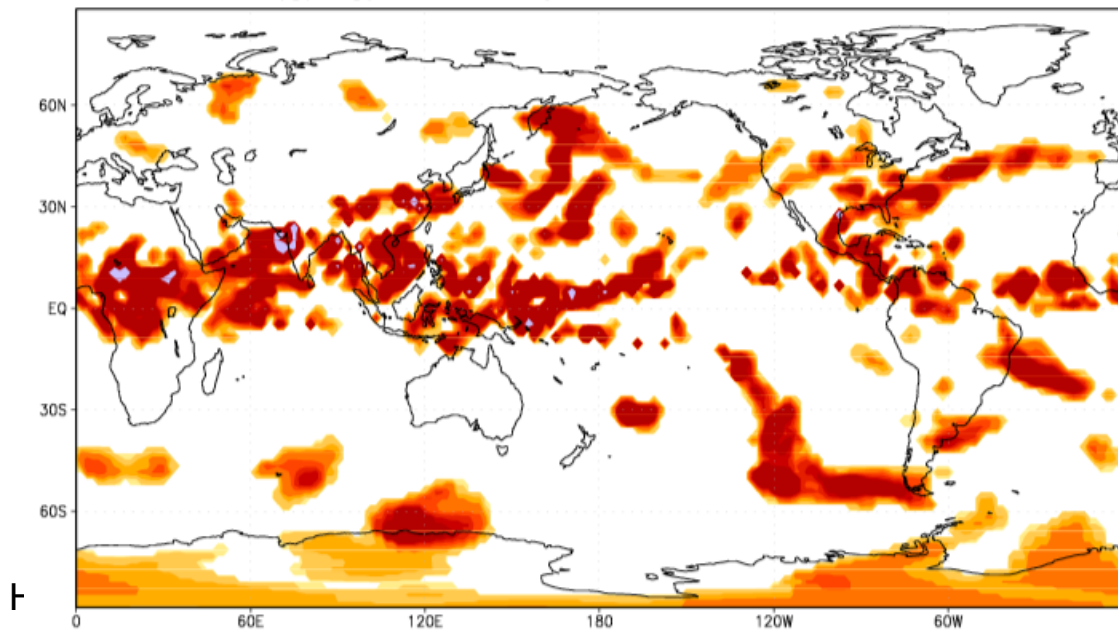
6hr fcst cloud water  
at model layer 35

nislfr

CLW(g/kg) model layer 35 hour 06 control run



CLW(g/kg) model layer 35 hour 06 with nislfr

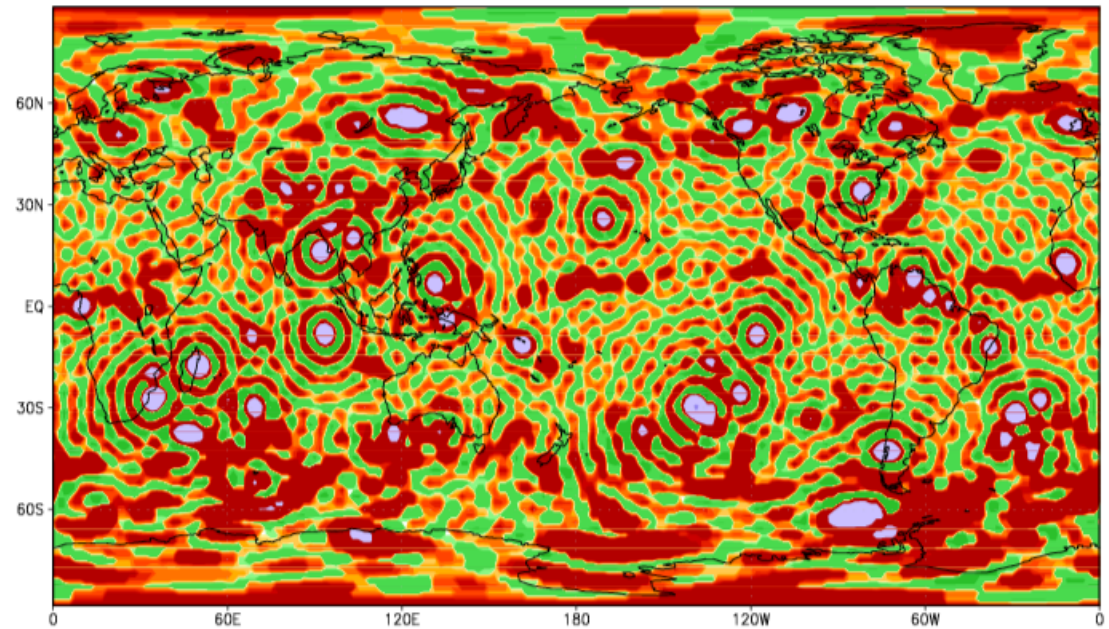


control

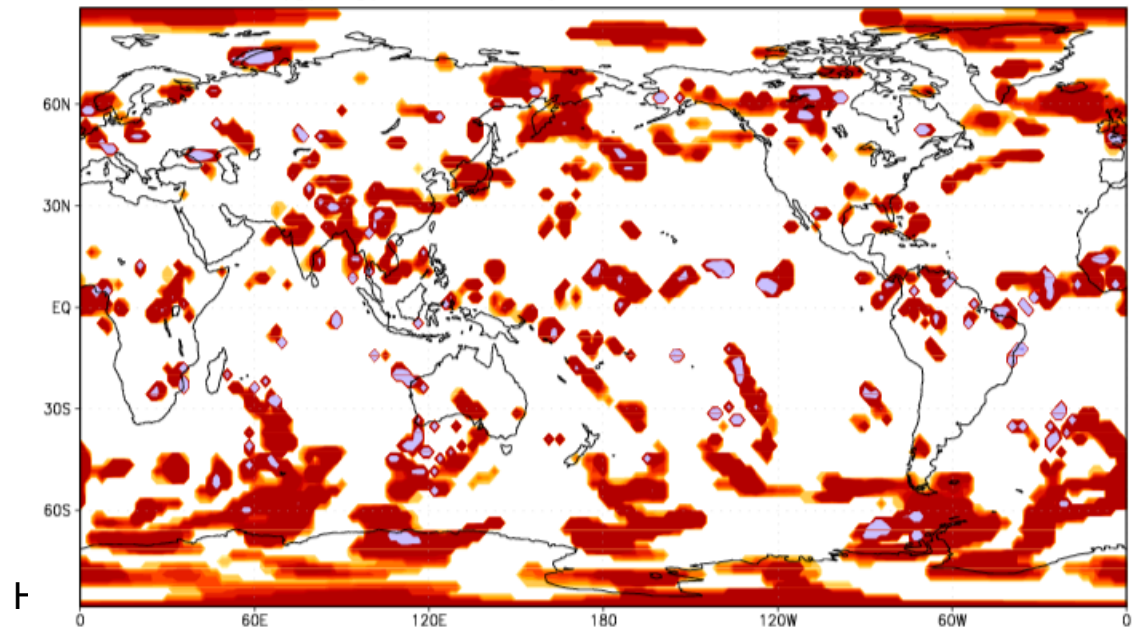
24hr fcst cloud water  
at model layer 30

nislfv

CLW(g/kg) model layer 20 hour 24 control run



CLW(g/kg) model layer 20 hour 24 with nislfv



1D SWE on spherical coordinates can be written as

$$\frac{\partial u}{\partial t} + u \frac{1}{a \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{g}{a \cos \phi} \frac{\partial H}{\partial \lambda} = 0$$
$$\frac{\partial h}{\partial t} + u \frac{1}{a \cos \phi} \frac{\partial h}{\partial \lambda} + \frac{h}{a \cos \phi} \left( \frac{\partial u}{\partial \lambda} \right) = 0$$

where

$$u = a \cos \phi \frac{d\lambda}{dt} \qquad H = h + h_s$$

which can be simplified as

$$\frac{\partial}{\partial t} \begin{pmatrix} h \\ u \end{pmatrix} + \begin{pmatrix} u & h \\ g & u \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} h \\ u \end{pmatrix} = 0$$

## Using diagonalization

$$L^{-1} \begin{pmatrix} u & h \\ g & u \end{pmatrix} L = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} \quad \& \quad LL^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

the equation

$$\frac{\partial}{\partial t} \begin{pmatrix} h \\ u \end{pmatrix} + \begin{pmatrix} u & h \\ g & u \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} h \\ u \end{pmatrix} = 0$$

can be

$$\frac{\partial}{\partial t} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} + \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = 0 \quad \text{or} \quad \frac{\partial R_i}{\partial t} + C_i \frac{\partial R_i}{\partial x} = 0 \quad \text{or} \quad \frac{dR_i}{dt} = 0$$

where

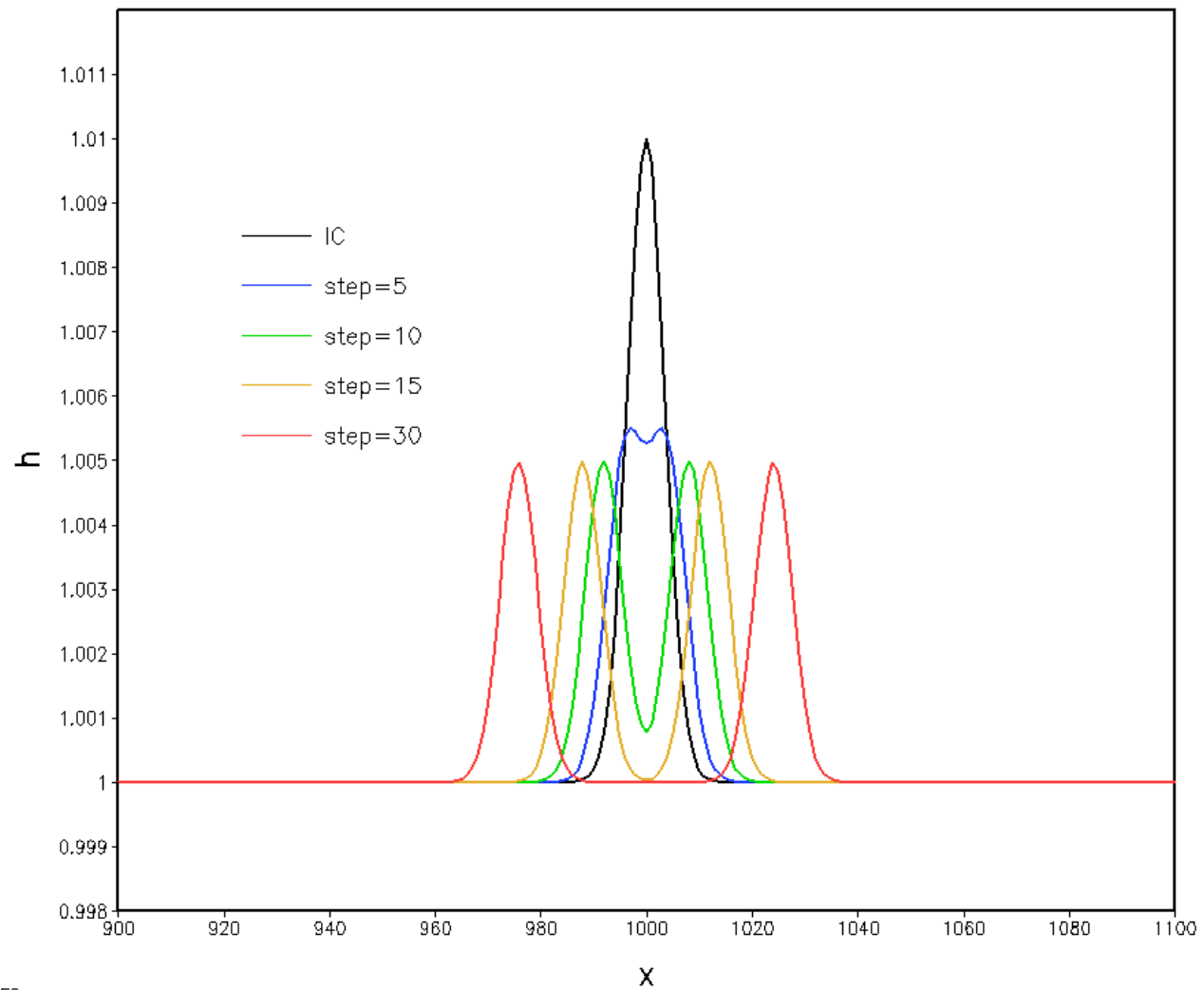
$$R_1 = \sqrt{gh} + u/2$$

$$R_2 = \sqrt{gh} - u/2$$

$$C_1 = u + \sqrt{gh}$$

$$C_2 = u - \sqrt{gh}$$

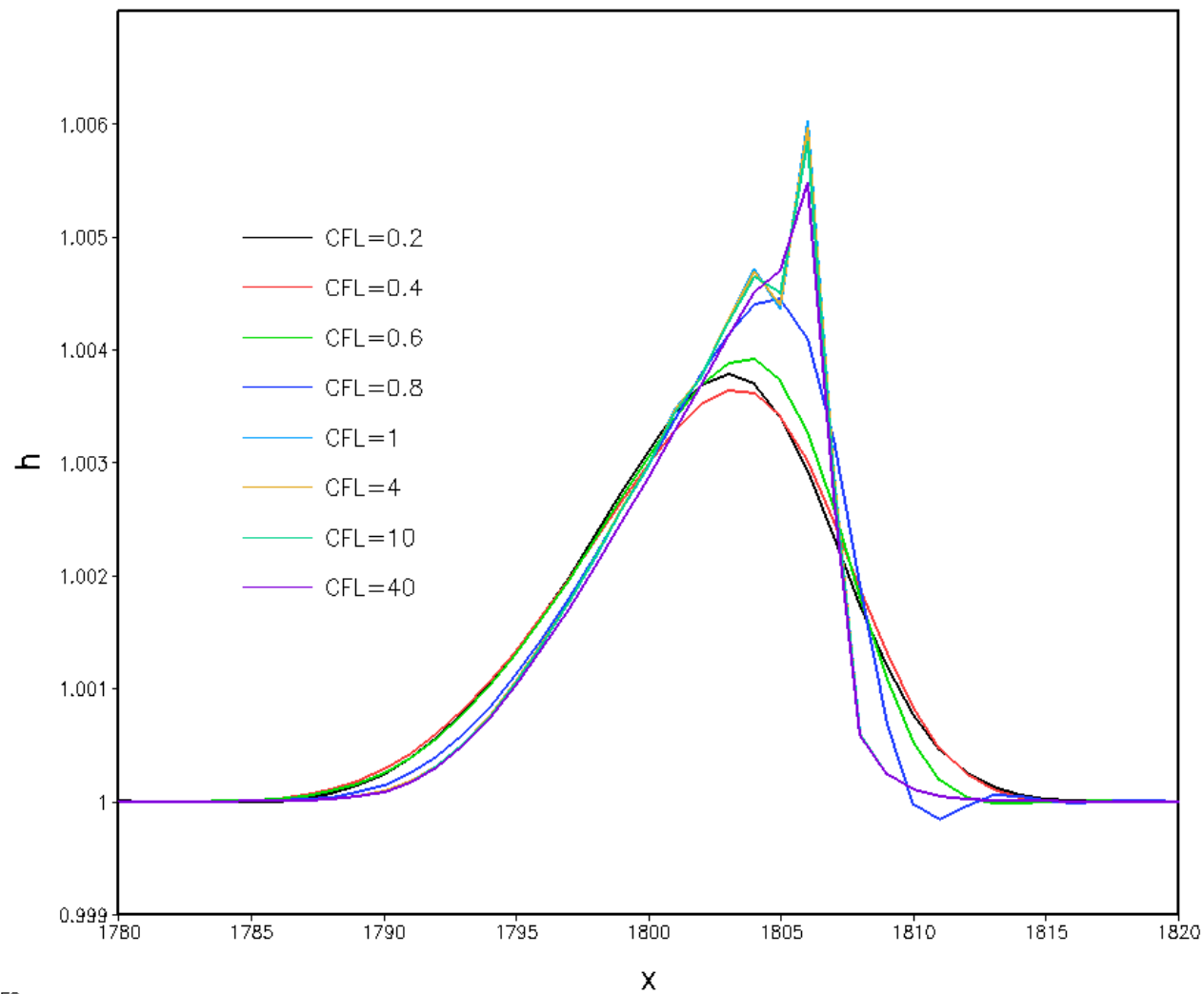
$h(x)$ , CFL=0.8



GrADS: COLA/IGES

Henry Juang - RSM2010

$h(x)$ ,  $t=800s$



GrADS: COLA/IGES

Henry Juang - RSM2010

# Nonhydrostatic system

$$\begin{aligned}\frac{\partial u}{\partial t} &= -u \frac{\partial u}{\partial x} - RT \frac{\partial Q}{\partial x} \\ \frac{\partial Q}{\partial t} &= -u \frac{\partial Q}{\partial x} - \gamma \left( \frac{\partial u}{\partial x} \right)\end{aligned}$$

For Riemann solver, we let the above be

$$\frac{\partial}{\partial t} \begin{pmatrix} Q \\ u \end{pmatrix} + \begin{pmatrix} u & \gamma \\ RT & u \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} Q \\ u \end{pmatrix} = 0$$

$$\begin{aligned}\frac{\partial \mathcal{R}_1}{\partial t} &= -c_1 \frac{\partial \mathcal{R}_1}{\partial x} \\ \frac{\partial \mathcal{R}_2}{\partial t} &= -c_2 \frac{\partial \mathcal{R}_2}{\partial x}\end{aligned}$$

or

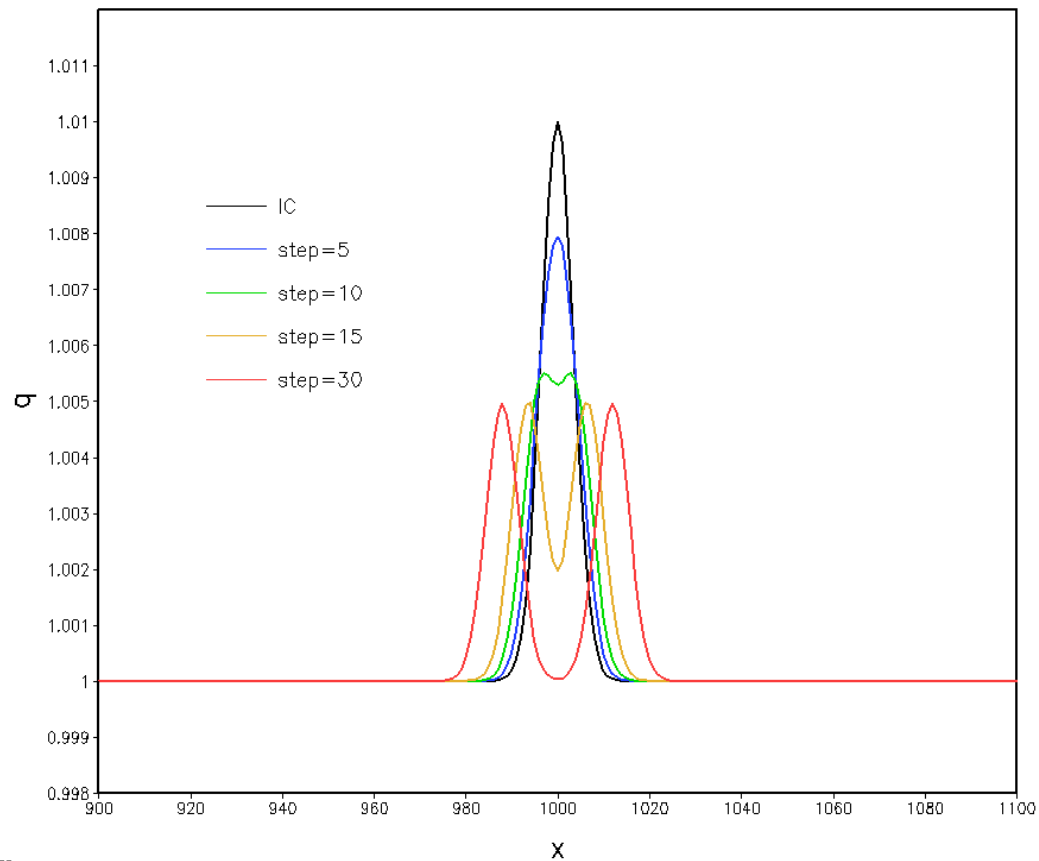
$$\begin{aligned}\frac{dR_1}{dt} &= 0 \\ \frac{dR_2}{dt} &= 0\end{aligned}$$

where

$$\begin{aligned}R_1 &= \sqrt{RT/\gamma} Q + u \\ R_2 &= \sqrt{RT/\gamma} Q - u \\ c_1 &= u + \sqrt{\gamma RT} \\ c_2 &= u - \sqrt{\gamma RT}\end{aligned}$$

# The initial acoustic spread

$q(x)$ , CFL=0.8

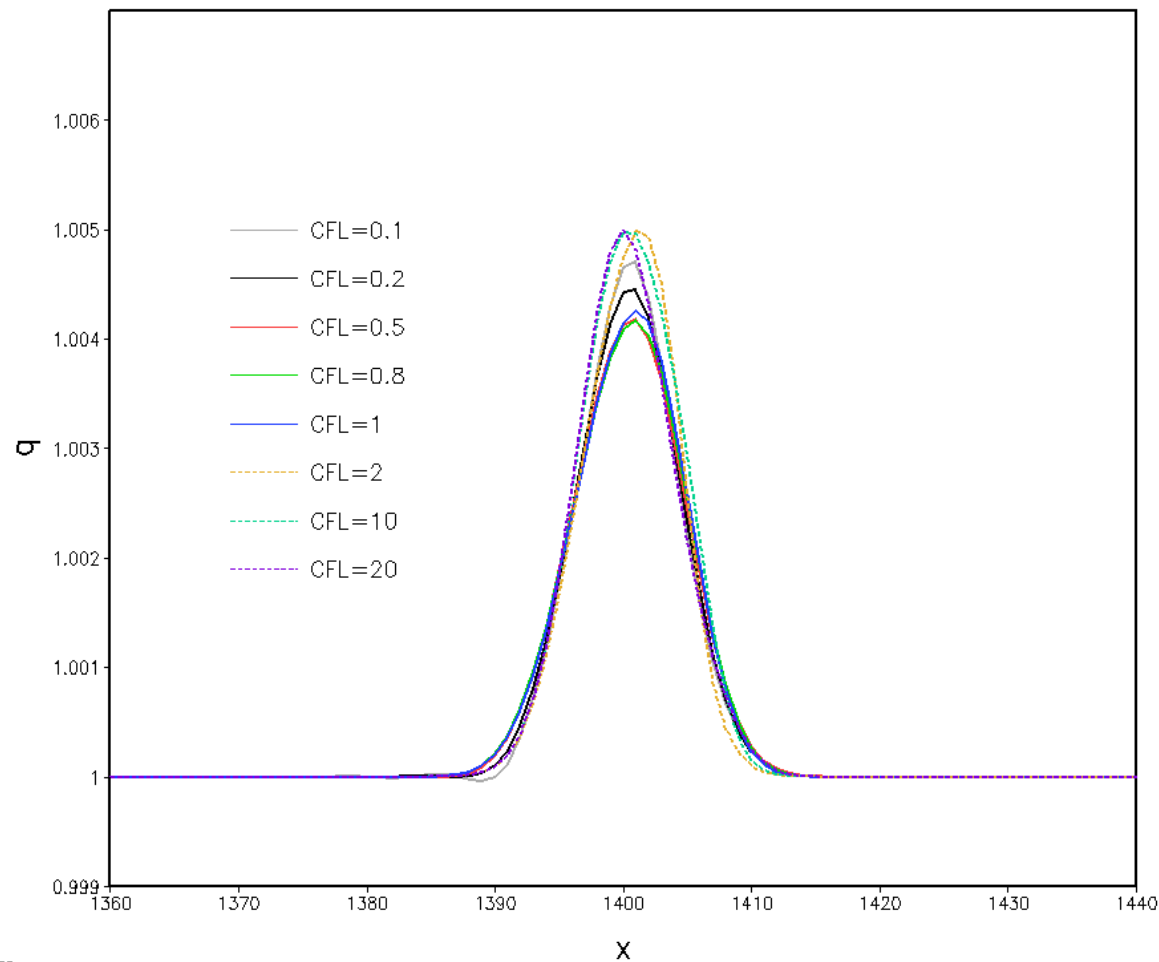


GrADS: COLA/IGES



# After 800s with different CFL

$q(x)$ ,  $t=800s$



## 2D nonhydrostatic tests in x-z with isotherm

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - w \frac{\partial u}{\partial z} - R\bar{T} \frac{\partial Q'}{\partial x}$$

$$\frac{\partial w}{\partial t} = -u \frac{\partial w}{\partial x} - w \frac{\partial w}{\partial z} - R\bar{T} \frac{\partial Q'}{\partial z}$$

$$\frac{\partial Q'}{\partial t} = -u \frac{\partial Q'}{\partial x} - w \frac{\partial Q'}{\partial z} - \gamma \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) + \frac{gw}{R\bar{T}}$$

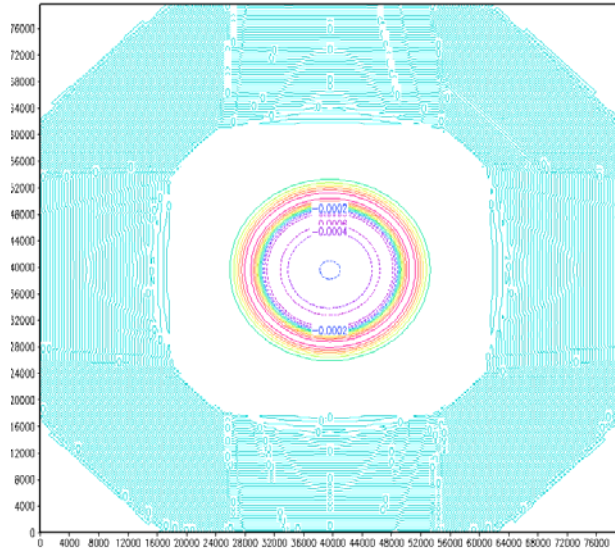
where

$$\frac{\partial \bar{Q}}{\partial z} = -\frac{g}{R\bar{T}}$$

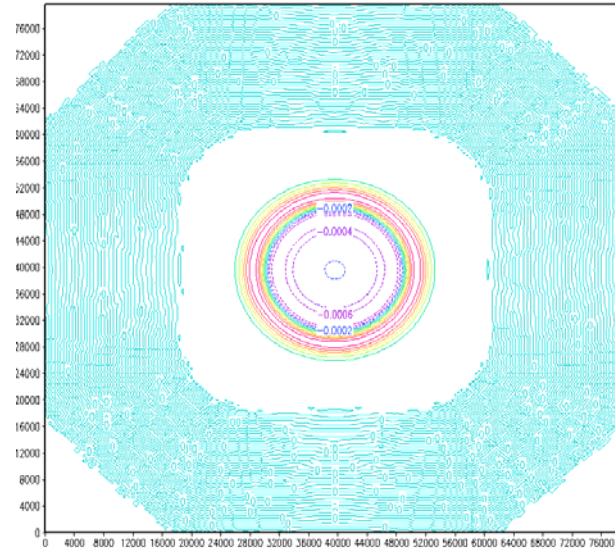
$$Q = \bar{Q} + Q'$$

# 2D tests in x-z (non-forcing) – $Q'(t=30s)$

Qpron ( $t=30s$ ,  $T_s=0.2$ ,  $CFL=0.35$ )

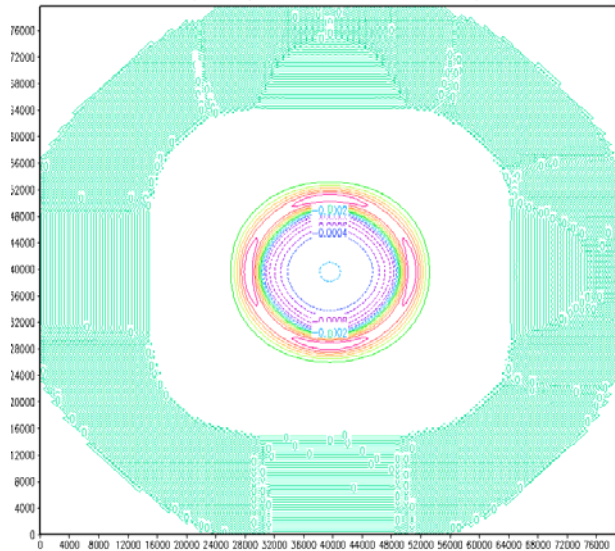


Qpron ( $t=30s$ ,  $T_s=0.5$ ,  $CFL=0.875$ )

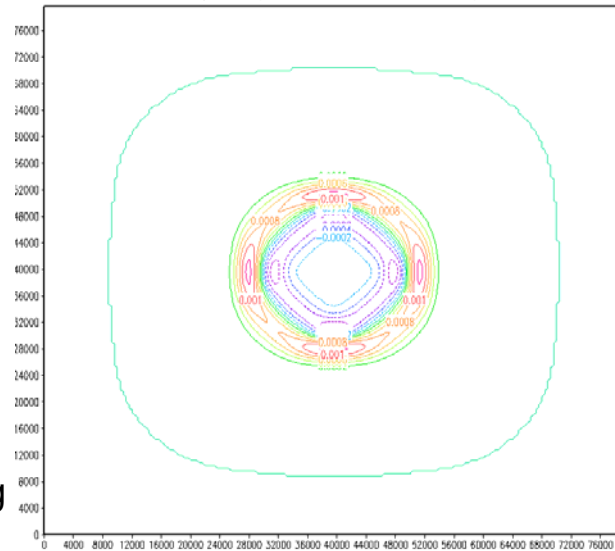


QADS: COLA/IGES

Qpron ( $t=30s$ ,  $T_s=1$ ,  $CFL=1.75$ )



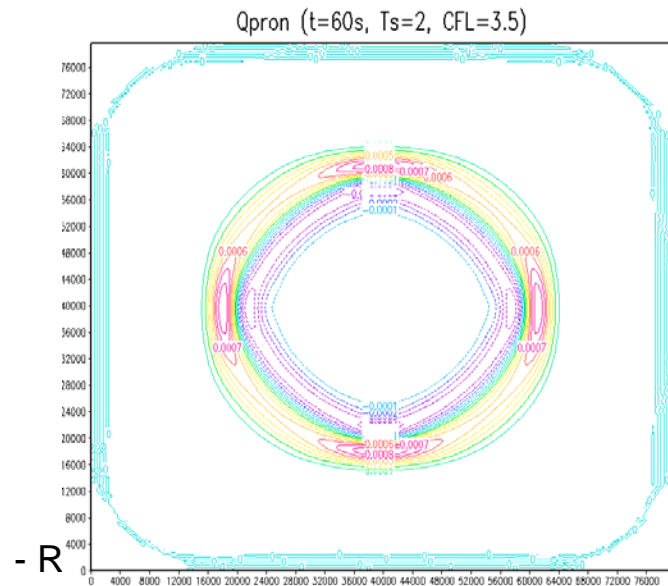
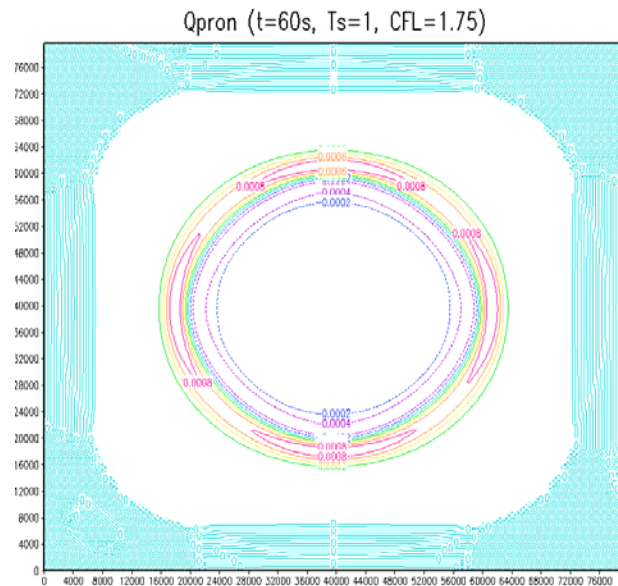
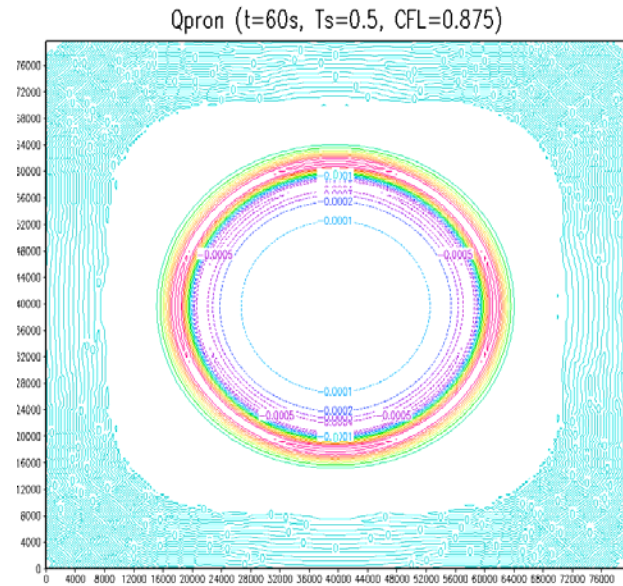
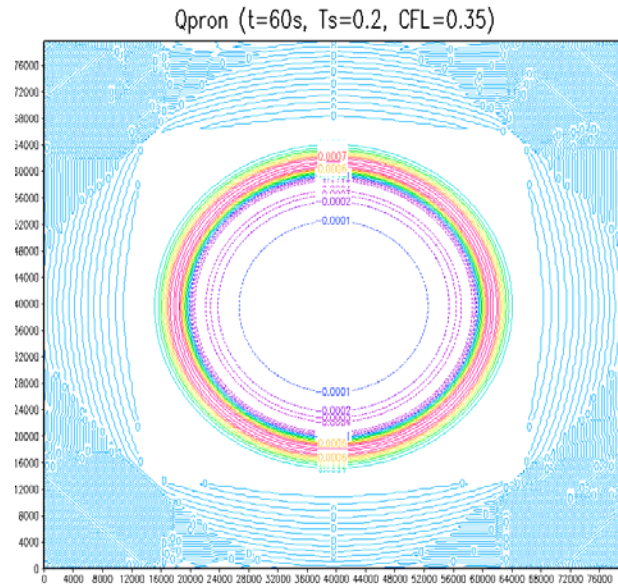
Qpron ( $t=30s$ ,  $T_s=2$ ,  $CFL=3.5$ )



QADS: COLA/IGES

ang

# 2D tests in x-z (non-forcing) – $Q'(t=60s)$



SWE on spherical coordinates can be written as

$$\frac{\partial u}{\partial t} + u \frac{1}{a \cos \phi} \frac{\partial u}{\partial \lambda} + v \frac{1}{a} \frac{\partial u}{\partial \phi} + \frac{g}{a \cos \phi} \frac{\partial H}{\partial \lambda} - \left( f + \frac{u \tan \phi}{a} \right) v = 0$$

$$\frac{\partial v}{\partial t} + u \frac{1}{a \cos \phi} \frac{\partial v}{\partial \lambda} + v \frac{1}{a} \frac{\partial v}{\partial \phi} + \frac{g}{a} \frac{\partial H}{\partial \phi} + \left( f + \frac{u \tan \phi}{a} \right) u = 0$$

$$\frac{\partial h}{\partial t} + u \frac{1}{a \cos \phi} \frac{\partial h}{\partial \lambda} + v \frac{1}{a} \frac{\partial h}{\partial \phi} + \frac{h}{a \cos \phi} \left( \frac{\partial u}{\partial \lambda} + \frac{\partial v \cos \phi}{\partial \phi} \right) = 0$$

where

$$u = a \cos \phi \frac{d\lambda}{dt}$$

$$v = a \frac{d\phi}{dt}$$

$$H = h + h_s$$

let

$$\frac{1}{a \cos \phi} \frac{\partial}{\partial \lambda} = \frac{\partial}{\partial \lambda'}$$

$$\frac{1}{a} \frac{\partial}{\partial \phi} = \frac{\partial}{\partial \phi'}$$

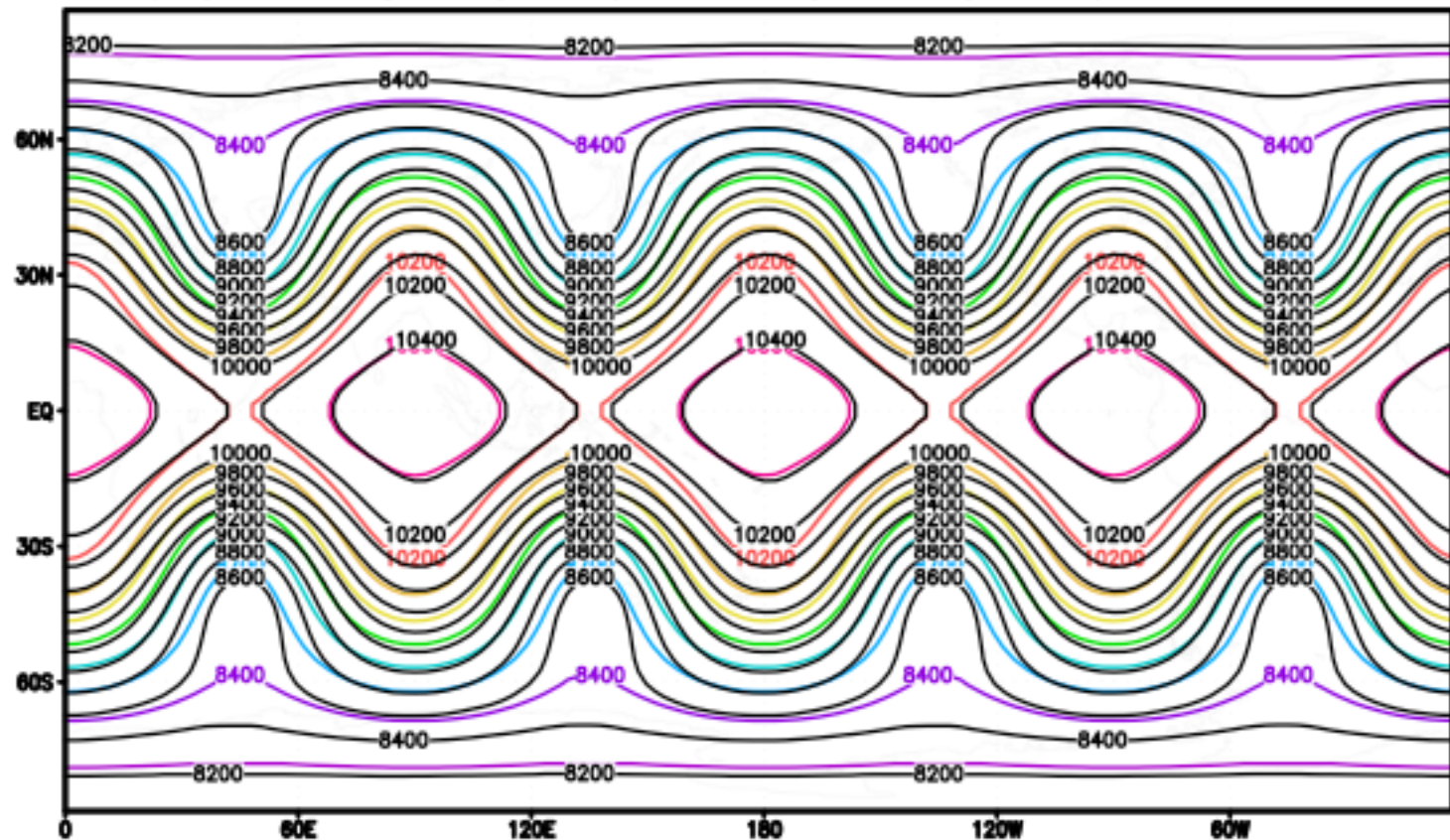
rewrite the previous SWE into

$$\frac{\partial}{\partial t} \begin{pmatrix} h \\ u \\ v \end{pmatrix} + \begin{pmatrix} u & h & 0 \\ g & u & 0 \\ 0 & 0 & u \end{pmatrix} \frac{\partial}{\partial \lambda'} \begin{pmatrix} h \\ u \\ v \end{pmatrix} + \begin{pmatrix} v & 0 & h \\ 0 & v & 0 \\ g & 0 & v \end{pmatrix} \frac{\partial}{\partial \phi'} \begin{pmatrix} h \\ u \\ v \end{pmatrix} + \begin{pmatrix} -\frac{\tan \phi}{a} h v \\ -f v - \frac{\tan \phi}{a} u v + g \frac{\partial h_s}{\partial \lambda'} \\ f u + \frac{\tan \phi}{a} u u + g \frac{\partial h_s}{\partial \phi'} \end{pmatrix} = 0$$

Then dimensional split into

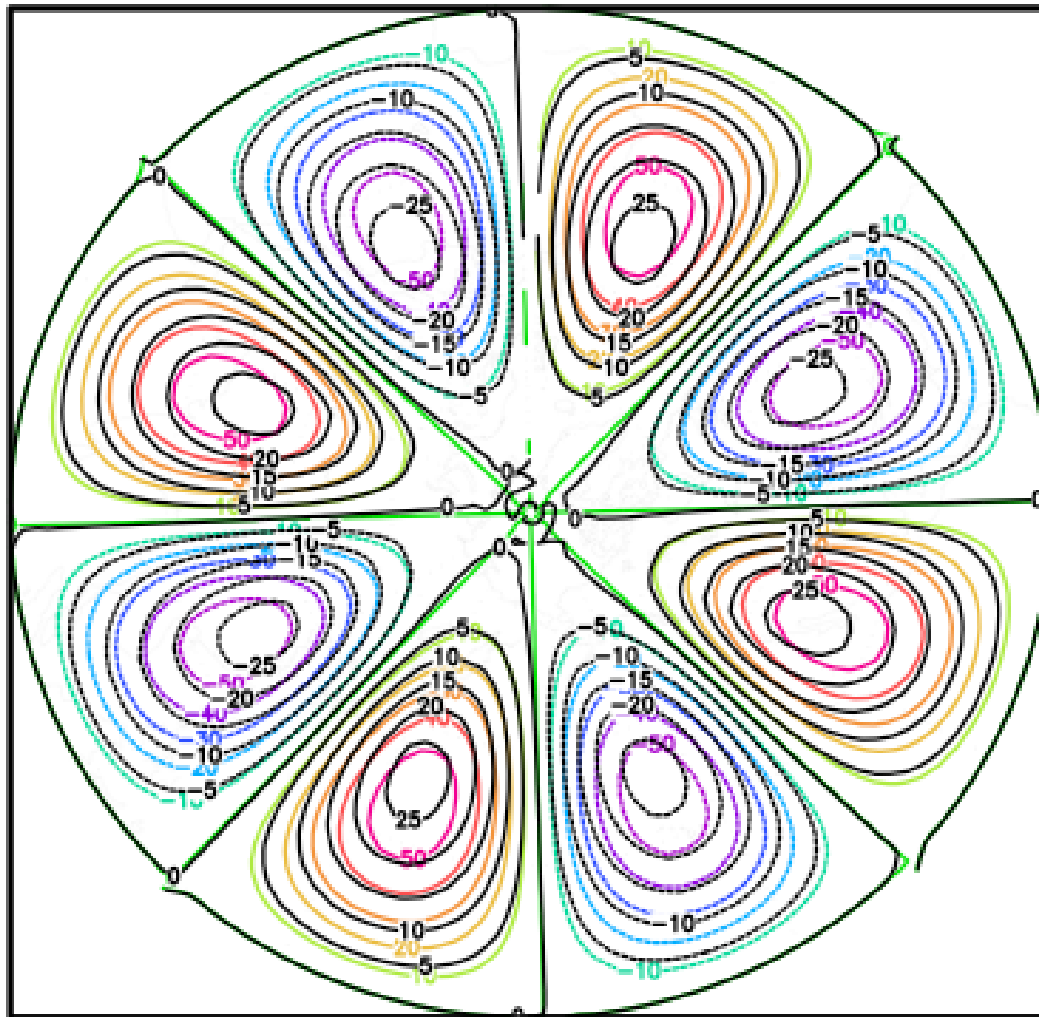
$$\begin{aligned} \frac{\partial}{\partial t} \begin{pmatrix} h \\ u \\ v \end{pmatrix} + \begin{pmatrix} u & h & 0 \\ g & u & 0 \\ 0 & 0 & u \end{pmatrix} \frac{\partial}{\partial \lambda'} \begin{pmatrix} h \\ u \\ v \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 \\ -f v - \frac{\tan \phi}{a} u v + g \frac{\partial h_s}{\partial \lambda'} \\ f u + \frac{\tan \phi}{a} u u + g \frac{\partial h_s}{\partial \phi'} \end{pmatrix} &= 0 \\ \frac{\partial}{\partial t} \begin{pmatrix} h \\ u \\ v \end{pmatrix} + \begin{pmatrix} v & 0 & h \\ 0 & v & 0 \\ g & 0 & v \end{pmatrix} \frac{\partial}{\partial \phi'} \begin{pmatrix} h \\ u \\ v \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -2 \frac{\tan \phi}{a} h v \\ -f v - \frac{\tan \phi}{a} u v + g \frac{\partial h_s}{\partial \lambda'} \\ f u + \frac{\tan \phi}{a} u u + g \frac{\partial h_s}{\partial \phi'} \end{pmatrix} &= 0 \end{aligned}$$

H(m) for day 0 (color) and day 7.9 (mono) 128x64x600





V(m/s) for day 0 (color) and day 25 (mono) 128x64x600





# Warm bubble case

- Non-hydrostatic equation on xz can be written as

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - w \frac{\partial u}{\partial z} - RT \frac{\partial Q'}{\partial x}$$

$$\frac{\partial w}{\partial t} = -u \frac{\partial w}{\partial x} - w \frac{\partial w}{\partial z} - RT \frac{\partial Q'}{\partial z} + g \frac{T'}{\bar{T}}$$

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - w \frac{\partial T}{\partial z} - \gamma RT \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right)$$

$$\frac{\partial Q'}{\partial t} = -u \frac{\partial Q'}{\partial x} - w \frac{\partial Q'}{\partial z} - \gamma \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) + \frac{gw}{RT}$$

$$\bar{T} = 303.16$$

$$\bar{\pi} = \left( \frac{\bar{p}}{p_0} \right)^{\frac{R}{C_p}}, \quad \bar{Q} = \ln \frac{\bar{p}}{p_0}$$

$$\frac{\partial \bar{\pi}}{\partial z} = -\frac{g}{C_p \bar{\theta}}, \quad \frac{\partial \bar{Q}}{\partial z} = -\frac{g}{R \bar{T}}$$

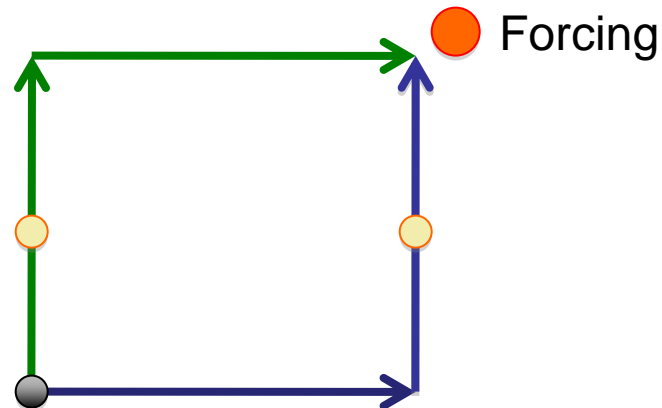
$$Q = \bar{Q} + Q', \quad T = \bar{T} + T'$$

# Warm bubble case

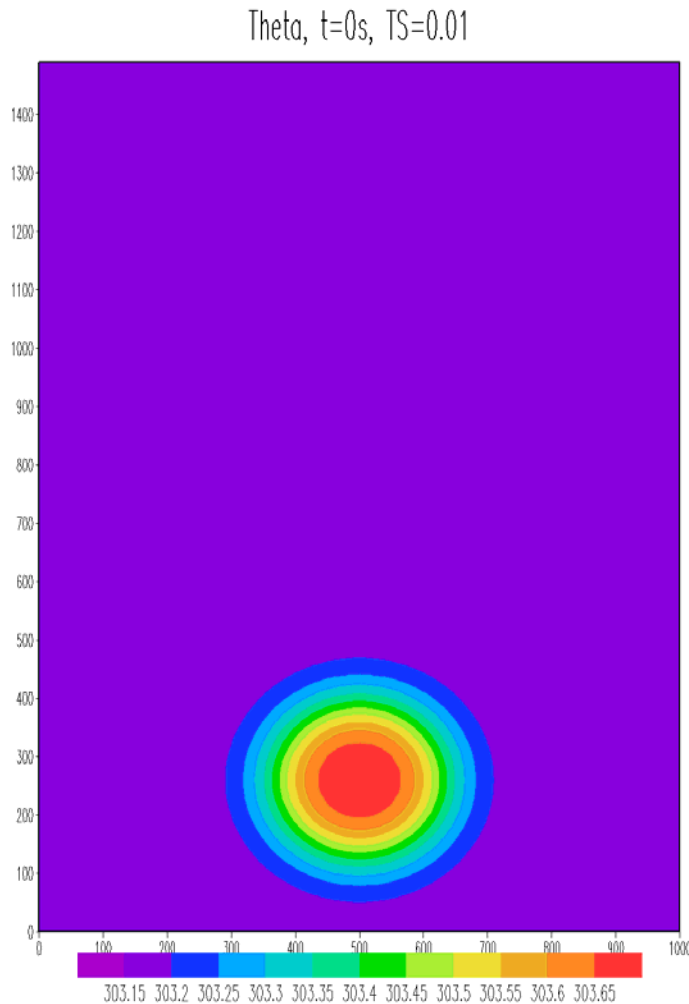
- For Riemann solver, we write it into

$$\frac{\partial}{\partial t} \begin{pmatrix} Q' \\ u \\ w \end{pmatrix} = - \begin{pmatrix} u & \gamma & 0 \\ R\bar{T} & u & 0 \\ 0 & 0 & u \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} Q' \\ u \\ w \end{pmatrix} - \begin{pmatrix} w & 0 & \gamma \\ 0 & w & 0 \\ R\bar{T} & 0 & w \end{pmatrix} \frac{\partial}{\partial z} \begin{pmatrix} Q' \\ u \\ w \end{pmatrix} + \begin{pmatrix} \frac{gw}{R\bar{T}} \\ -RT' \frac{\partial Q'}{\partial x} \\ -RT' \frac{\partial Q'}{\partial z} + g \frac{T'}{\bar{T}} \end{pmatrix}$$

$$\frac{\partial \theta}{\partial t} = -u \frac{\partial \theta}{\partial x} - w \frac{\partial \theta}{\partial z}$$



# Warm Bubble case



## Experiment setting

### Domain:

$$dx = dz = 10m$$

$$grids : 101 * 150$$

$$Bubble\ center(51,27)$$

### Initial Condition:

$$\bar{T}(i,j) = 303.16$$

$$u(i,j) = 0$$

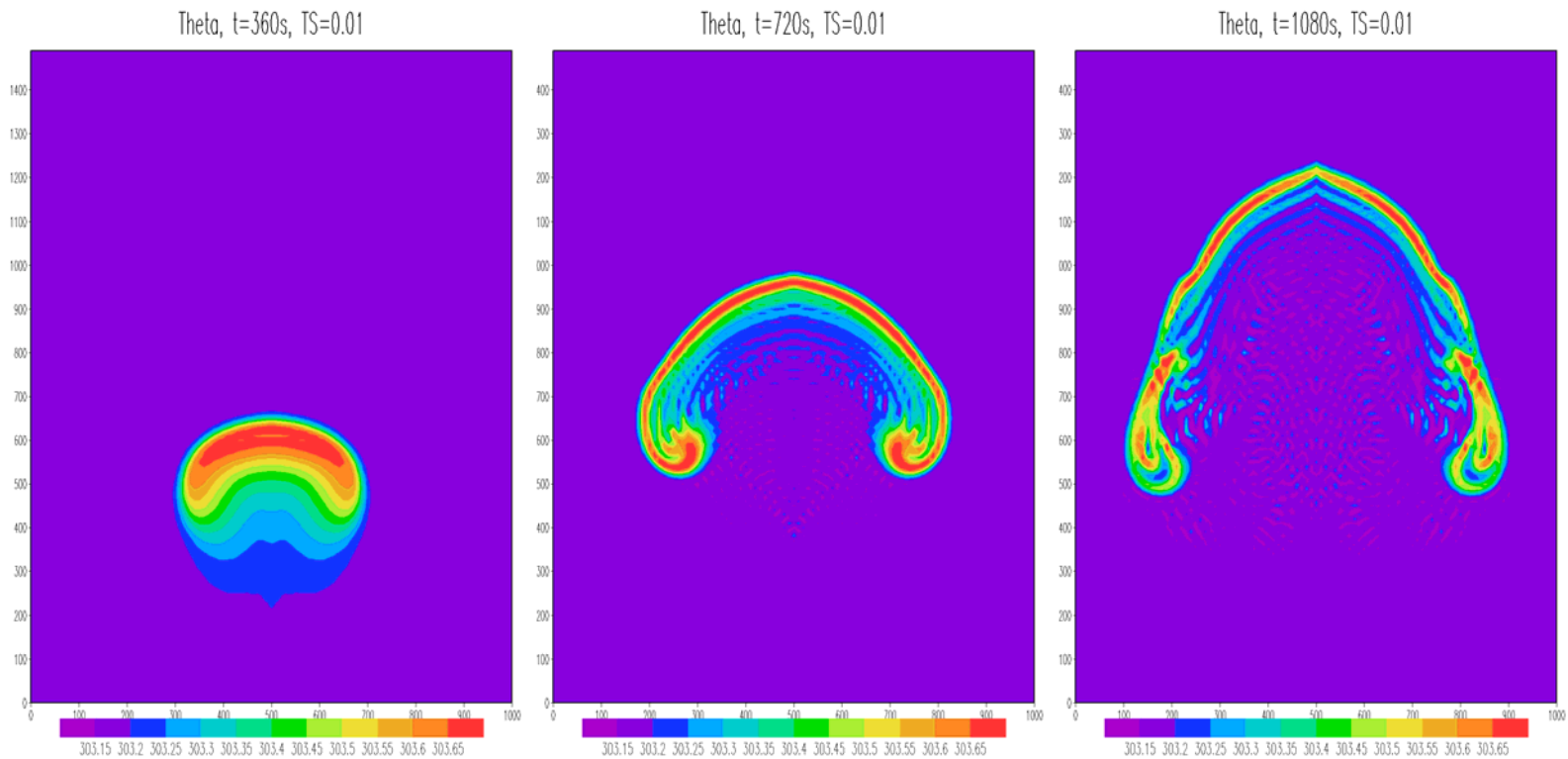
$$w(i,j) = 0$$

$$\theta(i,j) = \begin{cases} A \\ Ae^{-(r-a)^2/s^2}, r > a \end{cases}$$

$$r^2 = (x - x_0)^2 + (z - z_0)^2, a = 50, s = 100$$

# Warm Bubble Case

$\theta$  Time-step=0.01s, CFL~0.7



# The possible future

- More generalized and/or hybrid system will be built, more accurate thermodynamics will be used.
- Semi-Lagrangian will become more simple, conserving and economical code for time integration.
- Gravity and acoustic waves can be resolved explicitly.
- Deep atmosphere or non-approximated system will be introduced for further accuracy in model dynamics.

# Implementation

- New version will be implemented into NCEP version first
- NCEP code is managed under SVN (subversion)
- Will release periodically to upgrade other related version
- To simplify file structure, for users, we have only /SYS/ and /EXP/.

# NCEP SVN system directory

After port from NCEP svn version, all system can be under /SYS/ as

/doc/	faq installation txt pdf documents
/lib/	w3/ bacio/ sp/ etc
/utl/	rsmmap.sh etc
/src/	all source code, such as rinp, rmtn, rsm
/fix/	constant files
/jsh/	main scripts for run
/ush/	sub scripts for jsh
/inp/	example inputs
/exp/	examples for running RSM/MSM
/pak/	package check out

## NCEP SVN user directory

After initial setup the system in /SYS/, you can make your own directory as /EXP/, under /EXP/ you copy experimental example from /SYS/exp/, so you have

/EXP/gsmp2rsm/	get NOMAD pgb file to download
gsms2rsm/	get NOMAD sig/sfc files
rsm2msm/	get local rsm data to run msm

Each example directory has only 3 files

configure	to make your options
compile	use configure to compile all sources
run	use configure to run experiments